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Optimum sizing of residential cogeneration for prefeasibility estimations. An analytical approach

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Abstract

An analytical method is developed to estimate the optimum size of cogeneration units in residential applications used for electrical supply, space heating and domestic hot water production. The method is validated by comparing its results with those of detailed simulation from twenty-five main cities of Greece, and found to achieve a coefficient of variation of root-mean squared error less than 4%. The optimum sized CHP corresponds to 32-47% of maximum load, and proved to be mainly related to the heating degree-days of the area (correlation coefficient $R=0.985$) rather than the minimum temperature.

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1. Introduction

Combined heat and power production (CHP) may lead to significant primary energy savings and reduction of equivalent CO₂ emissions. Despite the attractive concept behind CHP, in practice there are a lot of barriers that restrict its application, with the most important probably being its high cost and the often limited utilization of the recoverable heat, especially when used in space heating applications. For this reason the size of the plant should be carefully selected, and in spite of the complexity of this issue, some guidelines have been proposed by suppliers like the specification at about 30-50% of the maximum thermal load (to cover 50-70% of annual thermal needs) and to achieve a minimum of 4000 hours operation annually [1], which are quite rough however.

Due to technical and economic limitations and the several parameters that affect the operation and the economy of the system, the selection of a CHP unit is actually based on a case by case optimization

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instead of a rule of thumb. Various optimization criteria are applicable to this aim, like the mean annual profit [2], the determination of the hourly trend of the daily thermal load [3], the total life cycle emissions [4] or a compromise of them through a multi-objective approach [5]. Although general purpose energy analysis software tools have been used for CHP evaluation (e.g. RETScreen [6]), dedicated simulation is necessary to accurately calculate the annual performance of the unit [7]. In the present work, a simplified model is firstly developed for the technical and economic prefeasibility evaluation and optimization of CHP in residential applications. In this context, an Internal Combustion Engine (ICE) based CHP is assumed. Typical electrical loads of dwellings and the use of recoverable heat for space heating and production of domestic hot water (DHW) are regarded. The analytical solution of the above optimization problem is developed and validated.

Nomenclature

C	specific cost (€/kWh)
n	efficiency
NB	net benefits (€)
P	capacity of the CHP unit (kW)
TLC	total heat loss coefficient of the dwelling (kW/K)
ΔT	heat demand intensity (K)

2. The model

A variety of CHP operation modes are applicable, falling into the main categories of heat match or electrical match. In general the heat match mode leads to the highest energy and cost efficiency in buildings, supplying the base thermal load through co-generative ICE and modulating the load with the use of auxiliary boilers. A heat match operation is consequently assumed here, and the specific net benefits per thermal energy unit recovered are:

$$SNB = \frac{n_{EL}}{n_{TH}} \cdot (C_{EL} - C_{MAIN}) - C_{GAS} \cdot \left[\frac{1}{n_{TH}} - \frac{1}{n_B} \right] \quad (1)$$

The efficiencies n_{EL} and n_{TH} vary with the load of the unit, and the same happens with the specific net benefits. The annual net benefits NB_{TOT} arise as the integral:

$$NB_{TOT} = 24 \cdot (TLC) \cdot \int_0^{365 \text{ days}} SNB \cdot \min\{\Delta T_d, \Delta T^+(t)\} \cdot dt \quad (2)$$

The CHP is assumed to cover the thermal needs of the dwelling up to $\Delta T_d = T_b - T_a$, where T_b is the base temperature of the dwelling and T_a the ambient temperature. The thermal capacity of the unit will be consequently $P_{TH} = (TLC) \cdot \Delta T_d$, and the electrical capacity $P_{EL} = P_{TH} \cdot (n_{EL}/n_{TH})$. An increase in heat demand intensity by $\Delta T_{DHW} = (Q_{DHW}/TLC)$ can be introduced for the addition of the thermal needs Q_{DHW} for domestic hot water production (DHW), and then a modified equivalent base temperature arises

$T_b^o = T_b + \Delta T_{DHW}$. Last, economic evaluation may be based on various economic indexes, like the benefit to cost ratio $BCR = NB_{TOT} \cdot PWF(N, r) / COST$, where $PWF(N, r)$ is the present worth factor, N the life time of the unit, r the discount rate and $COST$ is the cost of the unit installed.

Due to scale economy, specific capital expenses per installed capacity decrease when ΔT_d increases. On the other hand, the specific capital expenses per thermal energy recovered increase with ΔT_d due to the consequent decrease of load factor, hence the need for optimization. Considering that the various units have quite similar efficiencies, then the only optimization parameter is ΔT_d (or equivalently the size of the unit), which is easily calculated by e.g. a trial and error process.

3. Analytical approach

The accuracy of a quadrant relation of heating degree-days $DD_H(T) = a \cdot T^2 + b \cdot T + c$, was elsewhere documented [8,9]. For the estimation of coefficients a, b (which are needed for the calculations that follow), the mean temperature of the area T_{MEAN} and the heating degree-days $DD_H(T_{REF})$ at any reference temperature T_{REF} are only needed. To this aim, an apparent minimum temperature $T_{MIN,APP}$ should be firstly estimated:

$$T_{MIN,APP} = T_{REF} - \frac{2 \cdot DD_H(T_{REF})}{365} \cdot \left\{ 1 + \sqrt{1 - \frac{365 \cdot (T_{REF} - T_{MEAN})}{DD_H(T_{REF})}} \right\} \quad (3)$$

$$DD_H(T) = \left[\frac{91.2}{T_{MEAN} - T_{MIN,APP}} \right] \cdot T^2 + \left[\frac{-182.5 \cdot T_{MIN,APP}}{T_{MEAN} - T_{MIN,APP}} \right] \cdot T + \left[\frac{91.2 \cdot T_{MIN,APP}^2}{T_{MEAN} - T_{MIN,APP}} \right] \quad (4)$$

It is easily proved that the heat demand intensity curve results from the derivative of heating degree-days: $t = d[DD_H(T)]/dT = 2 \cdot a \cdot T + b = 2 \cdot a \cdot (T_b^o - \Delta T) + b$. The annual net benefits NB_{TOT} can be numerically estimated. By dividing ΔT_d to the intervals defined between the end-points $\varepsilon_k \cdot \Delta T_d$ where $0 = \varepsilon_{M+1} < \varepsilon_M < \dots < \varepsilon_k < \varepsilon_{k-1} < \dots < \varepsilon_1 = 1.0$, for $k=1$ to M , and by replacing the corresponding values of t_k in eq. (2) we get:

$$NB_{TOT} = 24 \cdot (TLC) \cdot a \cdot \left\{ SNB_1 \cdot \Delta T_d \cdot \left[2 \cdot T_b^o - (\varepsilon_1 + \varepsilon_2) \cdot \Delta T_d + \frac{b}{a} \right] + \sum_{k=2}^M [SNB_k^+ \cdot \varepsilon_k \cdot (\varepsilon_{k-1} - \varepsilon_{k+1}) \cdot \Delta T_d^2] \right\} \quad (5)$$

Assuming that the CHP cost follows an exponential relation $COST = A \cdot P_{EL}^q$ then the objective function is formulated to maximize the quantity:

$$MAX \left\{ SNB_1 \cdot \Delta T_d^{1-q} \cdot \left[2 \cdot T_b^o - (\varepsilon_1 + \varepsilon_2) \cdot \Delta T_d + \frac{b}{a} \right] + \sum_{k=2}^M [SNB_k^+ \cdot \varepsilon_k \cdot (\varepsilon_{k-1} - \varepsilon_{k+1}) \cdot \Delta T_d^{2-q}] \right\} \quad (6)$$

Condition of Eq. (4) is easily solved analytically, to get the optimum ΔT_d :

$$\Delta T_{d,opt} = \left(2 \cdot T_b^o + \frac{b}{a}\right) \cdot \left(\frac{1-q}{2-q}\right) \cdot \left(\frac{SNB_1}{SNB_1 \cdot (\varepsilon_1 + \varepsilon_2) - \sum_k [\varepsilon_k \cdot (\varepsilon_{k-1} - \varepsilon_{k+1}) \cdot SNB_k^+]}\right) \quad (7)$$

As a consequence, the optimum CHP unit has an electrical capacity $P_{EL} = \{TLC \cdot \Delta T_{d,opt} \cdot n_{EL}/n_{TH}\}$. Heat produced by the CHP unit is calculated again from eq. (5) by inserting the unit 1.0 in the place of SNB_1, SNB_k^+ . Electricity produced is also calculated with an equation similar to eq. (5) by inserting in the place of SNB_1, SNB_k^+ the ratios $(n_{EL,k}/n_{TH,k})$.

Notably, the method is still applicable when CHP operates a few hours per day (e.g. if switched off at low electricity tariffs periods). A heat demand intensity curve is drawn for the rest hours, and a best line is fitted $\Delta T = A \cdot t + B$. When t is introduced in days, the coefficients a and b are: $a = -1/(2 \cdot A)$, $b = (T_b^o - B)/A$, and the same eq. (1) and eq. (7) are also applied.

4. Demonstration of analytical approach - Case study

A case study is elaborated to demonstrate the analytical approach. Efficiency values and cost data are based on a market study included in [10]. For the relatively narrow range of interest, efficiencies are quite similar (see Fig. 1a, correlation coefficient almost zero). In the contrary, efficiency varies significantly with the part load (also shown in Fig. 1a). The cost of the CHP units varies with the capacity according to a scale exponent in the range of 0.45-0.50 (Fig. 1b). According to this figure, we assume the relation $COST = 10,200 \cdot P_{EL}^{0.45}$, where P_{EL} is introduced in kW and the cost is estimated in (€).

Temperature data are presented in Fig. 2.a, while all other data are presented in Table 1. According to Table 1 and eq. (3) & (4) it is estimated $T_{MIN,APP}=7.40^\circ\text{C}$, $a=8.524$, $b=-126.22$. Variation of electrical and thermal efficiencies of the CHP unit is further given in Table 2 (expressed on heat capacity basis), together with the calculations. It finally results $\Delta T_{d,opt}=7.37^\circ\text{C}$. For a two-storey dwelling with total $(TLC)=2.4$ kW/K the optimum CHP plant will consequently have a thermal capacity of 17.68kW_{th} and a respective electrical capacity of 7.22kW_e. The cost of the unit is estimated at 24,830€. The net annual benefits are estimated at 744€ and for $PWF(20,0.05)=12.46$ it is $BCR=0.37$ which is very low and means that the unit will not be paid back. The heat supplied is estimated at 64,470kW_{th} and the electrical supply at 25,120kW_e. The CHP plant could be specified to supply thermal needs for DHW, too, which are estimated at 5,000kWh annually. This means a mean load of $Q_{DHW}=5000/8760=0.57$ kW. The base temperature is correspondingly amended to $T_b^o = T_b + (Q_{DHW}/TLC) = 20 + 0.57/2.4=20.24^\circ\text{C}$. In this case it is estimated $\Delta T_{d,opt}=7.51^\circ\text{C}$, and the heat supply by the CHP unit increases to 66.950kW_{th}, which means that the CHP plant will cover almost 50% ($=100 \cdot [66,950 - 64,470]/5000$) of DHW needs.

Table 1. Data for the Case Study

Quantity	Value	Quantity	Value
Cost of electricity	0.162 €/kWh _e	Area	Athens
Cost of gas	0.076 €/kWh	Mean temperature	18.1°C
Cost of maintenance	0.025 €/kWh _e	Heating degree-days at 15.5°C	559.5 K-days
Electrical efficiency of CHP unit	24.3%	Base temperature	20°C
Thermal efficiency of CHP unit	59.5%	Boiler efficiency	90.0%

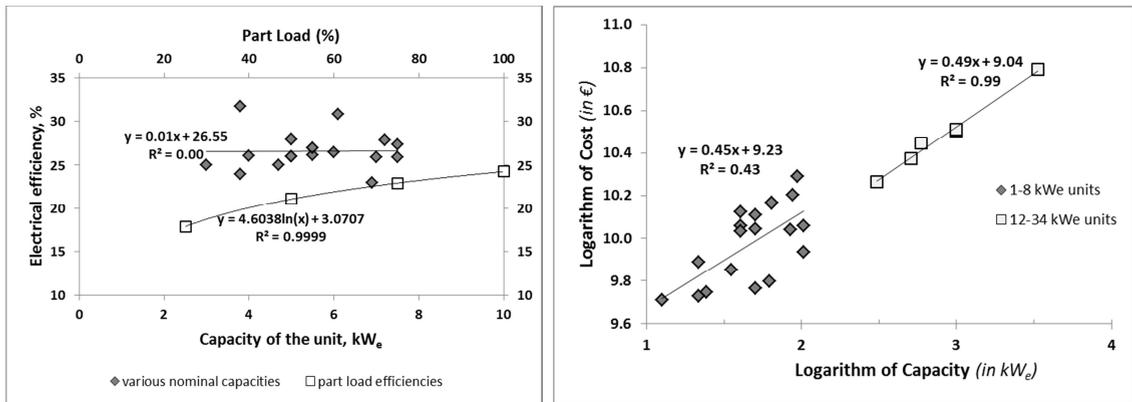


Fig. 1. (a) Variation of electrical efficiency of CHP units with the capacity (upper line) and with the load (lower curve) (b) Variation of purchase cost of CHP with the electrical capacity

Table 2. Calculations for the Case Study

k	LOAD (%)	ε_k	n_{EL}	n_{TH}	SNB_k	$SNB_k^+ \cdot \varepsilon_k \cdot (\varepsilon_{k-1} - \varepsilon_{k+1})$	$\varepsilon_k \cdot (\varepsilon_{k-1} - \varepsilon_{k+1})$	$\varepsilon_k \cdot (n_{EL,k} / n_{TH,k})$
1	100	1.00	0.243	0.595	0.01266	-	-	-
2	75	0.75	0.229	0.610	0.01129	0.00423	0.375	0.14078
3	50	0.50	0.211	0.621	0.00861	0.00215	0.250	0.08494
4	25	0.25	0.179	0.635	0.00331	0.00041	0.125	0.03524

5. Validation of the analytical approach

The analytical estimations of $\Delta T_{d,opt}$ elaborated for twenty-five main cities of Greece, are very close to the detailed simulation results (Fig. 2b), with a coefficient of determination factor $R^2=0.998$, mean bias error 0.13 and coefficient of variation of root-mean squared error CV(RMSE) of 3.6%, which is very satisfactory for prefeasibility analyses. According to these results, optimum ΔT_d ranges in the Country between 5.5-12.5°C, and proved to be better related to the heating degree-days of each area (linear correlation coefficient $R=0.985$) than its minimum temperature ($R=0.934$), as usually regarded. According to these results, optimum sized CHP covers 31.6-46.5 % of peak thermal load, which is in agreement with what CHP manufacturers recommend for peak load coverage between 30-50%. Obviously these results may be of limited importance due to the volatility of the data, but on the same reason does also spring the validity of the proposed algorithm.

6. Conclusions

Maximization of CHP load factor and scale economy are two competing factors making necessary the optimum sizing of CHP units. The analytical approach proposed in this work proved to be quite accurate, rendering optimum sizing of a CHP system a quick and straightforward process suitable for prefeasibility estimations.

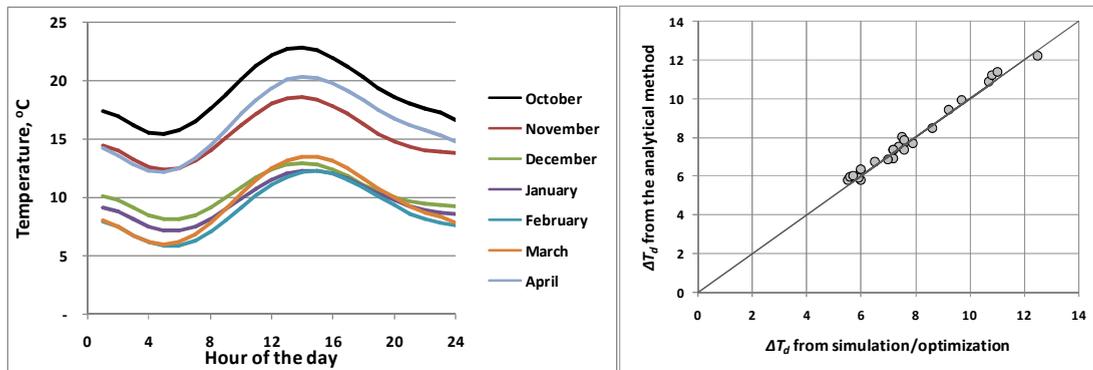


Fig. 2. (a) Ambient temperature data during heating period in Athens (b) Comparison between simulated and analytical results

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Biography

J.Gelegenis is a Professor at the Energy Engineering Technology Department, TEI of Athens. Previously he worked in the Greek *Center for Renewable Energy Sources (CRES)*, as a Sales Manager in *Technava S.A.* Contractors and Equipment Suppliers and he was the principal of *New Energy Techniques Engineering Consultancy* firm.