# MINIMIZATION OF OPERATIONAL COSTS IN COOLING WATER SYSTEMS

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In this work, an optimization model for a cooling water system which supplies a heat exchanger network is developed. The model considers the thermal and hydraulic interactions in the process, and is applied to the study and analysis of typical operational cases. The objective is to minimize the total operating cost of the system, which includes cooling water make-up and energy costs. A base case with fixed configuration is optimized for a given set of climatic conditions. Optimal operating conditions are obtained for several thermal specifications on the water that leaves the tower under different cost coefficients for water and energy. Additionally, constraints on the tower performance are imposed on the system as well as climatic changes. Results indicate that forced water withdrawal is an optimal operational practice to relieve the cooling tower load whenever additional heat must be removed from the cooling tower.

Keywords: cooling water systems; cooling towers; nonlinear optimization; process analysis; heat exchanger network; operational cost

# 1. INTRODUCTION

Cooling towers are usually present wherever water is used as a cooling medium. In wet or evaporative towers the water to be cooled comes in contact with the outside air. They are extensively used in many industries such as electricpower generation stations, refrigeration or air conditioning systems, and chemical and petrochemical process plants. Compared to dry towers where the water to be cooled flows within a finned surface over which atmospheric air is blown, the advantages of evaporative cooling towers include higher rates of heat and mass transfer per unit volume combined with a relatively low pressure drop and low initial and operational costs<sup>1</sup>.

The cooling process is accomplished by a combination of sensible heat transfer due to temperature difference and the evaporation of a small portion of the water. The second mechanism accounts for about 80% of the total heat removed<sup>2, 3</sup>.

Water cooling towers are sized and selected based on economic considerations as well as constraints imposed by system components. However, the thermal performance must ensure a precise cooling water temperature. This variable plays a pivotal role in most industrial applications and slight deviations from design specifications may have a significant impact on overall plant economics. Chemical plants establish their cooling water temperature on the operating pressures of the condenser of distillation and evaporation units, and consequently, on equipment preceding them. During the condensation process, the colder the condensing water, the higher the unit production and the lower the unit cost<sup>1</sup>. The importance of colder water for gas compression is also evident, since a major portion (nearly 80%) of energy converted to heat must be continuously removed at the same rate it is generated, or the compressor would overheat or shut down<sup>4</sup>. Again, in power generation plants, the temperature of cooling water sets the ultimate heat recovery from the turbine and the discharge pressure of the heat engines<sup>3</sup>.

Therefore, there has been a growing interest on cooling tower analysis. In the literature, many of the available studies have been concerned with the design of cooling towers<sup>2,5,6,7,8,1</sup>. Other studies have presented operation and control topics of cooling towers<sup>9,10,11,3,12,13</sup>. Additional studies have considered the mathematical modelling and simulation of thermal performance of cooling towers<sup>14,15,16,17,18,19</sup>. Moreover, Cheremisonoff and Cheremisinoff <sup>20</sup> compiled an extensive list of references concerning cooling tower design and operation.

However, despite the present widespread and continually growing interest in cooling tower rating, to the authors' knowledge, investigations concerning a systemic analysis and overview are not yet available.

In this work, an optimization model for a cooling water system which supplies a heat exchanger network is developed. The model considers the thermal and hydraulic interactions in the overall process and is applied to the analysis of typical operational cases. The system behaviour is studied under normal operating conditions and then compared to disturbances such as additional cooling load or colder temperatures requirement from the tower.

# 2. PROCESS DESCRIPTION

# 2.1. Cooling Tower Operation

The cooling water system considered in this work is shown schematically in Figure 1. It is a closed loop consisting of a cooling tower unit, a water circulation pump, an air blower and a heat exchanger network.



Figure 1. Cooling water system.

The performance of a cooling tower is measured by how close it brings the cold water temperature to the wet-bulb temperature of the surrounding air. The lower the wet-bulb temperature (which indicates either cool air, low humidity or a combination of both), the colder the tower can make the water. Nevertheless, the desired temperature of the cooled water is essentially greater than the wet-bulb temperature of the air. The difference between these temperatures is called 'approach' and its value<sup>13</sup> generally falls between 5°F and 20°F.

Another performance parameter is the difference between the temperature of the hot water entering the cooling tower and the temperature of the colder water leaving the tower, usually referred to as 'range'.

The thermal performance of a cooling tower depends on the packing arrangements as well as the circulating water and air flowrates. 'Rating factor' represents the number of tower units required for a given water rate and set of temperature conditions, usually expressed<sup>10</sup> in  $ft^2/gpm$ . A typical rating chart of a water cooling tower is shown in Figure 2.

During operation there is some loss of water. Firstly, water vapour passes through the cooling tower and is discharged into the atmosphere. Another source of water loss is due to entrained water droplets that escape from the tower with the exhaust air. Water is also lost from intermittent purge of small amounts of circulating water to prevent an increase in the concentration of solids due to evaporation. Make-up refers to the water flowrate required to replace the circulating water that is lost by evaporation, drift and blow-down.

Occasionally, forced withdrawal of circulating water, upstream of the tower, may be imposed to relieve the heat load so as to achieve lower temperatures on the water stream that leaves the tower with the corresponding make-up.

Since air flow promotes evaporation in the tower, an increase in air throughput constitutes another expedient way to increase the cooling capacity. Tower air flowrate can be controlled in several ways: on-off fans operation, use of variable-speed fans, use of automatically adjustable pitch fans<sup>13</sup>.

# 2.2. Process Interactions

The main process variables concerning the heat exchanger network are the individual cooling requirements,

the inlet and outlet temperatures of each cooler, the circulating pump performance and the split of flows through each branch. Due to strong interactions in the system, slight deviations from design specifications intervene on the overall plant behaviour.

The total circulating water flowrate depends on the pump operation point defined by its performance and the hydraulic characteristics of the system. The water flowrate through each branch of the pipe-cooler network is related to its individual flow resistance, given by the control valve adjustment and other fixed characteristics. However, any change in flowrate disturbs not only the flowrate through the pipe-cooler branches but also the cooler's outlet temperatures. This in turn affects the temperature of hot water at the inlet of the cooling tower.

In fact, the thermal behaviour of the system is even more



Figure 2. Rating chart for a typical cooling tower.

គម្ព័ង Εğ b. São Paulo - SP a.Porto Alegre-RS c. Manaus - A Legend Average maximum dry Average wet Average maximum wet bulb temperatures bulb temperatures temperatures Figure 3. Annual temperature charts. complex. Aside from process disturbances, the cooling

tower performance is influenced by climatic fluctuations (wet-bulb temperature or humidity of the ambient air throughout the year). This effect can be seen in Figure 3 through the climatic charts of three cities with very different temperature patterns along the year<sup>21</sup>. So, the actual tempera-ture of the cooled water will vary in accordance with these inevitable oscillations and it also affects the overall system.

Therefore, a realistic prediction of the operational conditions can only be done through a systemic analysis and overview, as will be seen in the remainder of this paper.

#### 3. MATHEMATICAL MODEL

In this section the main model constraints as well as the objective function are described. The following convention has been adopted: the functional relationships among the variables are represented in parentheses while brackets indicate the algebraic operations.

#### 3.1. Model Constraints

Cooling tower: A detailed phenomenological model of a cooling tower may become extremely complex. On the other hand, it is possible to establish correlation functions among the variables involved in typical performance diagrams, such as the one shown in Figure 2. Thus, in general form, for a given tower, one can write:

Range = Range (Approach,  $T_{wb}$ , rf) (1)

where:

$$Range = T_o - T_e \tag{2a}$$

$$Approach = T_e - T_{wb} \tag{2b}$$

However, it is more convenient for calculation purposes, as it will be seen later (see Section 4.1), to rearrange equation (1) so that the dependence of the process variables becomes explicit:

$$T_e = T_e(T_o, T_{wb}, rf) \tag{3}$$

For a given tower in operation, it is possible to write relation (4) for the rating factor which relates operating conditions  $(rf, w'_{a})$  with design conditions  $(rf^{o}, w'^{o}_{a})$ 

$$rf = \frac{rf^o w_e^0}{w_e'} \tag{4}$$

The performance of a tower in operation, as mentioned, is affected by the water and air flowrates. The humid air flowrate can be calculated through a water mass balance, by relating the amount of water that evaporates and the air humidity, as seen in equation (5)

$$v_{air} = \frac{w_{evap}}{[1 - \mathcal{H}_{in}][\mathcal{H}_{out} - \mathcal{H}_{in}]}$$
(5)

The absolute humidity of the air entering the tower is calculated at the ambient temperature:

$$\mathcal{H}_{in} = -\frac{c_s}{\lambda_w} [T_{amb} - T_{wb}] + \mathcal{H}_w \tag{6}$$

Note that in (6) the absolute humidity  $\mathcal{H}_w$  is calculated at the wet bulb temperature as well as the enthalpy  $\lambda_{w}$ . It is assumed that the air that leaves the tower is at the saturation condition<sup>10</sup>. Thus:

$$\mathcal{H}_{out} = \frac{MW_w}{MW_{air}} \left[ \frac{P_{vap}}{P - P_{vap}} \right] \tag{7}$$

where  $P_{vap}$  is determined at the air outlet temperature, given by the average of the inlet and outlet temperatures of the water<sup>10</sup>

As described in Section 2.2, the make-up water must supply all the water which is lost in the system.

$$w_{mu} = w_{evap} + w_{rem} + w_{entr} + w_{purge}$$
(8)

It is important to note that the portion of the recycled water flowrate which is intentionally removed  $(w_{rem})$  is eliminated upstream of the tower (see Figure 1), in order to reduce its load. Thus, the amount of water cooled in the tower is given by:

$$w'_e = w_e - w_{rem} \tag{9}$$

where  $w_e$  is the water flowrate in the closed circuit.

The make-up water is at ambient temperature  $(T_{amb})$ and therefore affects the temperature of the circulating water  $(T'_{e})$ . This effect is more significant when the make-up flowrate is high, as seen in equation (10).

$$T'_{e} = \frac{w_{e}T_{e} - w_{mu}[T_{e} - T_{amb}]}{w_{e}}$$
(10)

According to Perry and Green<sup>22</sup>, the water flowrate that evaporates can be estimated as:

$$w_{evap} = 0.00153w'_{e}[T_{o} - T_{e}]$$
(11)

where the temperatures are expressed in °C.

The amount of water purged is established from the concentration cycles, defined as the ratio between the amount of solids dissolved (mostly chlorides) in the recycled water and in the make-up water<sup>23</sup>. The number of concentration cycles is expressed as<sup>22</sup>:

$$n_{cycles} = \frac{w_{purge} + w_{evap}}{w_{purge}}$$
(12)

Rearranging equation (12) yields:

$$w_{purge} = \frac{w_{evap}}{n_{cycles} - 1} \tag{13}$$



The water loss by entrainment is assumed to be 0.1% with respect to the water flowrate through the tower<sup>22</sup>.

$$w_{entr} = 0.001 w'_{e} \tag{14}$$

*Pump*: The total pressure drop in the circuit  $(\Delta P_{total})$  comprises the contribution from the common line  $(\Delta P_{line})$  and the contribution of one of the individual branches which are in parallel, for instance the pressure drop in branch 1,  $\Delta P_1$ .

$$\Delta P_{total} = \Delta P_{line} + \Delta P_1 \tag{15}$$

Since the cooling tower operates at atmospheric pressure, this pressure variation corresponds to its increase in the circulation pump. From the characteristic curve of the pump, given in general form by equation (16), its operation point can be determined.

$$\Delta P_{total} = \Delta P_{total}(w_e) \tag{16}$$

*Common line*: The height variation between the pump outlet and the cooling tower inlet is neglected, since it is a value which is fixed in the problem. Then, the pressure drop in the external loop can be given as:

$$\Delta P_{line} = \frac{2\rho f_{line} L_{eq,line} v_{line}^2}{D_{line}}$$
(17)

where

$$v_{line} = \frac{4w_e}{\rho \pi D_{line}^2} \tag{18}$$

$$f_{line} = f_{line} \left( \frac{\rho v_{line} \, D_{line}}{\mu_{line}}, e_{line} \right) \tag{19}$$

The friction factor can be calculated, for instance, from the equation proposed by  $Chen^{24}$ .

*Heat exchanger* + *individual branch*: The heat load  $Q_i$  to be removed by the cooling water is specified for each heat exchanger E - i.

$$Q_i = w_i c_p [T_{o,i} - T'_e] \qquad i = 1, \dots n$$
(20)

In the proposed model, neither the conditions of the process fluids nor the thermal performance of the heat exchangers, such as the influence of the water flowrate in the global heat coefficient, were considered explicitly. Nevertheless, the influence of the cooling water conditions on the system, such as specifications for the water outlet temperature and/or flowrate for a given heat exchanger (see Section 4.2) will be studied.

The total pressure drop in an individual branch consists of the pressure drop in the line and the pressure drop in the exchanger. The former is subdivided in a variable term (according to the valve opening) and a fixed part (pipe and fittings) while in the latter, there is a contribution from the straight tube as well as from the direction changes.

$$\Delta P_i = \Delta P_{1,i} + \Delta P_{t,i} \qquad i = 1, \dots n \tag{21}$$

By neglecting again the height variation between the extremes of the branches in parallel, equation (21) yields:

$$\Delta P_{i} = \frac{2\rho f_{i} [L_{eq,i} + L_{va,i}] v_{i}^{2}}{D_{i}} + \frac{\rho f_{t,i} L_{t,i} v_{t,i}^{2} n_{t,i}}{2D_{t,i}} + \frac{K_{t,i} v_{t,i}^{2} n_{t,i}}{2} \quad i = 1, \dots n$$
(22)

where

ρ

$$v_i = \frac{4w_i}{\rho \pi D_i^2} \qquad i = 1, \dots n \tag{23}$$

$$v_{t,i} = \frac{4w_i n_{t,i}}{\rho \pi D_{t,i}^2 N_{t,i}} \qquad i = 1, \dots n$$
(24)

$$f_i = f_i \left( \frac{\rho \, vi \, D_i}{\mu_i}, \, e_i \right) \qquad i = 1, \dots n \tag{25}$$

$$f_{t,i} = f_{t,i} \left( \frac{\rho v_{t,i} D_{t,i}}{\mu_i}, e_i \right) \qquad i = 1, \dots n$$
(26)

$$K_{t,i} = K_{t,i} \left( \frac{\rho v_{t,i} D_{t,i}}{\mu_i}, n_{t,i} \right) \qquad i = 1, \dots n$$
 (27)

Again, in order to obtain the friction factor in the pipes  $(f_i)$ , Chen's correlation can be used<sup>24</sup>. The tube side friction factor in the heat exchanger  $(f_{t,i})$  and the term related to the tube side return pressure loss  $(K_{t,i})$ , can be determined with the equations in Kern<sup>25</sup>.

As a consequence of the stream split in the parallel branches, the pressure drop in the branches must be the same, that is:

$$\Delta P_i = \Delta P_j \qquad i = 1, \dots, n, i \neq j \tag{28}$$

*Node*: The water streams that leave the branches are mixed before returning to the tower. The mass and energy balances in the mixing point are given respectively as:

$$w_e = \sum_{i=1}^n w_i \tag{29}$$

$$T_o = \frac{\sum_{i=1}^n w_i T_{o,i}}{w_o}$$
(30)

*Physical properties*: The density and specific heat of the cooling water are calculated at the inlet temperature in the heat exchanger branches. These properties are assumed constant due to their small variation in the range of the operating temperatures.

$$=\rho(T_{e}^{\prime})\tag{31}$$

$$c_p = c_p(T'_e) \tag{32}$$

The viscosity values in the common line are calculated at the average inlet and outlet tower temperatures and, in each branch, calculated at the average inlet and outlet heat exchanger temperatures.

$$\mu_{line} = \mu_{line} \left( \frac{T'_e + T_o}{2} \right) \tag{33}$$

$$\mu_{i} = \mu_{i} \left( \frac{T'_{e} + T_{o,i}}{2} \right) \qquad i = 1, \dots n$$
(34)

The analytical relations for the physical properties can be found, for instance, in Yaws<sup>26</sup>.

# 3.2. Objective Function

The objective function is the minimization of the overall operating cost (\$/unit time), given by adding the cost of electricity and the cost of cooling water. The former is composed of the pumping cost and the fan operating cost,

while the latter is related to the make-up water.

$$C_{tot} = C_{elec} + C_{cw} = C_{pump} + C_{fan} + C_{cw}$$
(35)

By defining  $c_{elec}$  as the cost coefficient for electricity (\$/unit energy) and  $c_{cw}$  as the cost coefficient for cooling water (\$/unit mass), function (35) becomes:

$$C_{tot} = c_{elec}[P_f + P_p] + c_{cw}w_{mu}$$
(36)

In equation (36), the make-up water flowrate is calculated from equation (8). The power consumed by the fan is a function of the air flowrate. An example is given by the following expression<sup>10</sup>:

$$P_f = 0.0548 \left[ \frac{w_{air}}{\rho_{air}} \right] \tag{37}$$

where  $P_f$  is given in W and the term in brackets is expressed in m<sup>3</sup> h<sup>-1</sup>; the air flowrate can be calculated through a water mass balance, as given in equation (5).

Finally, the term  $P_p$  is related with the characteristic curve of the pump (equation (16)) and its efficiency  $\eta_p$ . An example is given by equation (38):

$$P_{p} = 1283 \frac{1}{\eta_{p}} \left[ \frac{w_{e}}{\rho} \right]^{0.476}$$
(38)

where  $P_p$  is given in W and the term in brackets is again expressed in m<sup>3</sup> h<sup>-1</sup>.

#### 3.3. Optimization Model

The optimization model can be written as:

Minimize (36) subject to (3)-(11), (13), (14)cooling tower (15), (16)pump (17), (18), (19) common line  $(20), (22)-(27) \quad i = 1, \dots, n$ heat exchanger+ branch  $i = 1, \ldots, n - 1$  parallel branches (28)(29), (30)node (31), (32), (33)physical properties (34)  $i = 1, \ldots, n$ physical properties (37), (38)cost constraints

In the optimization model, there are 10n + 24 variables and 9n + 22 equality constraints with additional single bound constraints, where *n* is the number of branches in parallel. The optimization variables are the air flowrate in the cooling tower ( $w_{air}$ ), the forced withdrawal of water ( $w_{rem}$ ), and the equivalent length of the valves in each branch ( $L_{vai}$ , i = 1, ..., n).

The model was implemented in a spreadsheet system and solved with the GRG2 code<sup>27</sup>. The algorithm is based on the Generalized Reduced Gradient Method and the solver parameters utilized are the following: automatic scaling, forward difference derivative calculations, tangent estimates and conjugate search. The convergence tolerance is set to  $1 \times 10^{-4}$ .

# 4. CASE STUDIES

The optimization model presented in Section 3 is rather general, since in principle any cooling tower algebraic representation as well as pump curve could be fit into the model. In Section 4.1, a base case is optimized for a given system configuration and climatic conditions. The effect of the water outlet temperature on the optimal cost is considered under different cost factors in Section 4.2. Finally, the tower performance is studied for different monthly climatic conditions. It is important to note that although the results are derived for a set of conditions, most of the conclusions can be extended towards other systems.

#### 4.1. Base Case

A schematic diagram of the system is described in Figure 1, which is composed of five heat exchangers which use cooling water that circulates through the tower. The main specifications of the equipment units are given as follows.

The rating chart given in Figure 2 was used to represent the tower performance. The operating points were converted into two analytical relations:

$$\begin{split} T_{e} &= 2.39043. \ T_{o}^{0.391536}. \ T_{wb}^{0.411002}. \ rf^{-0.0738408} \ (39) \\ T_{e} &= - \ 3.30739 + 1.30112. \ T_{o} - \ 4.10190 \times 10^{-1}. \\ T_{wb} + 2.10315 \times 10^{1}. \ rf \\ &+ \ 2.55208 \times 10^{-3}. \ T_{o}^{2} + 1.49159 \times 10^{-2}. \\ T_{wb}^{2} - \ 2.93774 \times 10^{-1}. \ rf^{2} \\ &- \ 7.70596 \times 10^{-3}. \ T_{o} \cdot T_{wb} - \ 3.84507 \times 10^{-1}. \\ T_{o} \cdot rf - \ 1.86696 \times 10^{-1}. \ T_{wb} \cdot rf \\ &- \ 1.23738 \times 10^{-5}. \ T_{o}^{3} - \ 9.84468 \times 10^{-5}. \\ T_{wb}^{3} - \ 2.84486 \times 10^{-1}. \ rf^{3} \\ &- \ 2.82492 \times 10^{-5}. \ T_{o}^{2}. \ T_{wb} + \ 7.24055 \times 10^{-5}. \\ T_{o} \cdot T_{wb}^{2} \\ &+ \ 6.26771 \times 10^{-4}. \ T_{o}^{2}. \ rf + \ 3.143289 \times 10^{-2}. \end{split}$$

$$T_o \cdot rf^2$$
  
+ 2.23001 × 10<sup>-3</sup>.  $T_{wb}^2 \cdot rf$  + 6.01565 × 10<sup>-3</sup>.  
 $T_{vb} \cdot rf^2$ 

The temperatures are given in °F and the rating factor in ft<sup>2</sup>/gpm. The operating range is defined by the following bounds:  $65^{\circ}F \le T_e \le 98^{\circ}F$ ,  $71^{\circ}F \le T_o \le 138^{\circ}F$ ,  $60^{\circ}F \le T_{wb} \le 80^{\circ}F$  and  $0.5 \text{ ft}^2/\text{gpm} \le rf \le 4.0 \text{ ft}^2/\text{gpm}$ .

Equation (40) showed a higher correlation degree and therefore will be used in this work. However, the influence of  $T_o$ ,  $T_{wb}$  and rf on the tower performance can be better visualized from equation (39). Hence, the higher both the tower inlet temperature and the wet bulb temperature, the higher is the tower outlet temperature, with the same order of magnitude influence (similar exponents). On the other hand, a higher rating factor causes a decrease in the tower outlet temperature.

As for the periodical water purges, four concentration cycles were adopted<sup>22</sup>.

The circulating water flowrate at design conditions  $(w_e^{\prime o})$  was 200000 kg h<sup>-1</sup> which corresponds to a rating factor

Table 1. Data for the example	e (pipes and heat exchangers)
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Branch i	Pip	e <sup>(*)</sup>	Heat Exchanger						
	$L_{eq,i}$ (m)	$D_i$ (m)	E-i	$n_{t,i}$	$N_{t,i}$	$D_{t,i}$ (m)	$L_{t,i}$ (m)	$Q_i$ (W)	
1	20	0.1023	E-1	1	92	0.016	2.44	241028	
2	50	0.0627	E-2	2	30	0.016	2.44	76929	
3	50	0.0627	E-3	2	30	0.016	2.44	272629	
4	50	0.0627	E-4	2	30	0.016	2.44	232222	
5	50	0.0627	E-5	2	30	0.016	2.44	34833	

(\*) carbon steel.

 $(rf^{\circ})$  of 1. In this base case, there is no forced withdrawal of circulating water and thus the only make-up required is related to evaporated water, purge and drift.

The characteristic curve of the pump for circulating water corresponds to the one of the ETA 100-26 / KSB model.

With respect to the line common to all heat exchangers in the circuit, the total length adopted is 320 m, with internal diameter of 0.1541 m (6" Sch 40, carbon steel).

Data for the heat exchangers and the main specifications of the pipes in parallel are shown in Table 1.

The atmospheric conditions are as follows: dry and wet bulb temperatures of 23°C and 21°C respectively. These values correspond approximately to average annual conditions in Porto Alegre, Brazil (see Figure 3a).

The optimal results of this case are shown in Figure 4. Note that, since the amount of make-up water is not significant, the value of the water temperature that feeds the exchangers is approximately the same as the value of the stream that leaves the tower. The value of the approach in the tower is  $3.75^{\circ}$ C, with range of  $4.10^{\circ}$ C. The values of the flow velocity as well as the pressure losses in both pipes and heat exchangers are within the ranges recommended in the literature.

The apparently high value of the air flowrate through the

cooling tower (approximately  $140000 \text{ kg h}^{-1}$ ) is due to the unfavourable inlet air conditions (relative humidity above 80%).

#### 4.2. Influence of Cost Factors and Water Outlet Temperature

In this study, the influence of different temperature specifications of the water that leaves the tower in the optimal solution was considered. This is a practical constraint imposed by the process, for instance, by the operating pressures of a distillation column. Additionally, the effect of the water and electricity cost coefficients was analysed.

Figure 5 shows the total operating cost versus the temperature of the outlet stream. Note that the curves are parameterized by the cost coefficient of water and they all converge to the same point that corresponds to the case for which there are no constraints in the water outlet temperature (approximately 24.75  $^{\circ}$ C).

Note that in Figure 5 the lower the outlet temperature target, the more sensitive the total cost is to the water cost coefficient. This can be also seen in Table 2 which shows the values of the process variables for  $c_{cw} = 1$ . When no constraint is imposed on the temperature, the cost of cooling



Figure 4. Optimal solution for the base case.



Figure 5. Optimal costs for different cooling water cost coefficients and outlet temperature.

Table 2. Optimal values of process variables for constrained outlet temperatures.

$T_{e}$ (°C)	C <sub>cw</sub> (\$/h)	$C_{pump}$ (\$/h)	C <sub>fan</sub> (\$/h)	C <sub>tot</sub> (\$/h)	w <sub>e</sub> (kg/h)	w <sub>rem</sub> (kg/h)	$(kg/h)^{W'_e}$	w <sub>evap</sub> (kg/h)	w <sub>air</sub> (kg/h)	approach (°C)	range (°C)	$ \stackrel{T'_e}{(°C)} $	$\stackrel{T_o}{(^{\circ}\mathrm{C})}$	<i>rf</i> (ft²/gpm)
24.75	0.03	1.31	0.41	1.75	187362	0	187362	1176	142032	3.75	4.10	24.73	28.85	1.07
24.50	0.14	1.31	0.41	1.86	187342	7147	180195	1117	141834	3.50	4.05	24.43	28.55	1.11
24.00	0.39	1.31	0.41	2.10	187303	23661	163642	998	140662	3.00	3.99	23.87	27.99	1.22
23.50	0.69	1.31	0.39	2.40	187268	43729	143539	879	137356	2.50	4.00	23.38	27.50	1.39
23.00	1.09	1.31	0.37	2.76	187240	69464	117776	743	128274	2.00	4.12	23.00	27.12	1.70
22.50	1.88	1.39	0.31	3.58	211439	121124	90315	544	108911	1.50	3.94	22.79	26.44	2.21

water represents less than 2% of the overall operating cost; however, this cost reaches 53% when the outlet water temperature is set at  $22.50^{\circ}$ C.

For temperatures lower than 24.50°C, there is a forced

withdrawal of cooling water and further make-up flowrate, caused by the incapacity of the tower to provide the cooling requirements. Therefore, for higher water cost coefficients the higher is the influence in the total cost.

![](_page_6_Figure_8.jpeg)

Figure 6. Optimal costs for different electricity cost coefficients and outlet temperature.

![](_page_7_Figure_1.jpeg)

Figure 7. Optimal solutions along the year for different rating factors (Porto Alegre, RS).

Similar behaviour to that in Table 2 is observed for the remaining water cost coefficients. As the water flowrate through the tower decreases, it causes a reduction in the evaporated water flowrate (see equation (11)). Consequently, there is a smaller air flowrate in the tower. Also, the rating factor increases due to a smaller water flowrate. It is interesting to note that in the last entry of Table 2 the temperature of the water stream after make-up slightly increases, since in this case the ambient temperature ( $T_{amb} = 23^{\circ}$ C) is higher than the temperature of the water that leaves the tower.

The influence of the temperature of the water that leaves the tower under different electricity cost coefficients on the total cost is shown in Figure 6. Note that in this case the total cost is less sensitive to lower target temperatures of the outlet stream. This is due to the fact that the electricity cost component plays a more significant role in the total operating cost. The process variables present similar behaviour as that shown in Table 2.

### 4.3. Influence of Tower Performance and Monthly Climatic Conditions

In this study the climatological chart of Porto Alegre (shown in Figure 3a) was considered, due to the fact that it is a city with very well defined meteorological changes. Hence, for the same system described in the base case, the operation was optimized for the corresponding dry and wet bulb average temperatures along the months.

Figure 7 shows the total optimal cost as a function of the month as well as for constraints on the rating factor. As mentioned, the rating factor provides an estimate of the tower performance. In other words, for a given water flow-rate high values of the rating factor indicate that a large amount of area is required for cooling the water; on the other hand, low values of the rating factor point to a high tower performance. The unconstrained case corresponds to the point of minimal total cost, which corresponds to a rating factor of  $1.07 \text{ ft}^2/\text{gpm}$ . By imposing constraints on the rating factor, the results indicate an increase in the total cost due to different reasons which can be better seen by examining the main process variables for January shown in Table 3.

It is important to note that higher values of the rating factor, namely 1.2 and 1.5, require the forced withdrawal of water from the tower and subsequent addition of water at ambient temperature. This implies higher costs for water and therefore a significantly higher overall cost; in fact from the last entry of Table 3 it can be seen that the cooling water cost reaches approximately 34% of the overall cost. On the other hand, lower values of the rating factor correspond to a higher tower capacity, which elevates water and air flowrates. Interestingly, the temperatures of the water leaving the tower  $(T_e)$  are lower than the temperature of the water after make-up  $(T'_e)$ . This may seem

Table 3. Optimal values of process variables for constrained rating factors (January).

<i>rf</i> (ft²/gpm)	C <sub>cw</sub> (\$/h)	$C_{pump}$ (\$/h)	C <sub>fan</sub> (\$/h)	C <sub>tot</sub> (\$/h)	w <sub>e</sub> (kg/h)	w <sub>rem</sub> (kg/h)	w' <sub>e</sub> (kg/h)	w <sub>evap</sub> (kg/h)	w <sub>air</sub> (kg/h)	approach (°C)	range (°C)	$\overset{T'_e}{(^{^{\circ}\!\!\!C})}$	$(^{\circ}C)$	$(^{\circ}C)$
0.95	0.03	1.39	0.38	1.79	210526	0	210526	991	127527	2.72	3.68	28.74	32.65	28.72
0.99	0.03	1.36	0.37	1.76	202020	0	202020	1111	126646	2.68	3.83	28.70	32.64	28.68
1.04	0.03	1.33	0.37	1.73	192308	0	192308	1185	125433	2.63	4.03	28.65	32.66	28.63
1.07	0.03	1.31	0.37	1.71	187588	0	187588	1185	124773	2.61	4.13	28.63	32.74	28.61
1.2	0.35	1.31	0.36	2.02	187583	20916	166667	1185	122106	2.28	4.36	28.52	32.51	28.28
1.5	0.86	1.31	0.34	2.51	187584	54253	133331	1185	114248	1.80	4.86	28.54	32.40	27.80

![](_page_8_Figure_1.jpeg)

Figure 8. Optimal costs along the year (Porto Alegre, RS).

contradictory but, as mentioned, the main purpose of water forced withdrawal and further make-up is to increase the tower efficiency by reducing its load.

In Figure 7, one can observe that, for a given value of the rating factor, the highest cost corresponds to the month of July. This might seem in principle paradoxical, as it is the coldest month of the year (in Brazil). Note, however, that it is the month with the highest humidity which can be seen in Figure 3a (the smallest difference between the dry and wet bulb temperatures). This confirms the fact that the main mechanism for water cooling is evaporation.

The optimal values of the overall cost along the year are shown in Figure 8. The pumping cost as well as the water cost shows almost no change; however, there is an increase in the air blowing cost. It is interesting to note that the cost profile follows the humidity pattern shown in Figure 3a.

#### 5. CONCLUSIONS

As a result of the strong interaction among the several process variables involved, the operational analysis of their effects on a cooling water system is very complex. Besides, tight constraints on the temperature of the water stream that leaves the tower may increase the operating cost substantially, in particular for the case in which the cost of water is high. Finally, results obtained from different climatic conditions point to the fact that the most important influence on the cooling system performance is not the ambient temperature itself, but its humidity. In summary, the general trend observed is that forced withdrawal of water upstream of the tower is an important resource for fulfilling the cooling duty requirements.

# NOMENCLATURE

$C_{cw}$	cost coefficient for cooling water
$C_{elec}$	cost coefficient for electricity
$c_p$	specific heat of cooling water
Ċ.	specific heat of humid air

$C_{cw}$	cost of cooling water
Celec	cost of electricity
Cfan	fan operational cost
Crum	pump operational cost
C	total operating cost
$D_{in}$	pipe inside diameter of branch <i>i</i>
$D_l$	nine inside diameter of common line
$D_{line}$	tube diameter of heat exchanger in branch <i>i</i>
$E_{t,i}$	heat exchanger in branch <i>i</i>
	nicat exchanger in branch <i>i</i>
e <sub>i</sub>	pipe fugosity in blanch <i>i</i>
e <sub>line</sub>	Exprise friction factor in give of breach i
Ji	Faming friction factor in pipe of branch <i>i</i>
] line	Fanning inclion factor in common line
Ĵt,i	tube-side friction factor of heat exchanger in branch i
$\mathcal{H}_{in}$	absolute humidity of the air that enters the tower
$\mathcal{H}_{out}$	absolute humidity of the air that leaves the tower
$\mathcal{H}_w$	absolute humidity of the air at the saturation condition
$K_{t,i}$	tube-side return pressure loss of heat exchanger in branch <i>i</i>
$L_{eq,i}$	equivalent length in branch <i>i</i>
L <sub>eq,line</sub>	equivalent length in common line
$L_{t,i}$	tube length of heat exchanger in branch <i>i</i>
$L_{va,i}$	equivalent length of the valve in branch <i>i</i>
MWair	molecular weight of the air
$MW_w$	molecular weight of the water
n	number of parallel branches in the system
neveles	number of concentration cycles
n <sub>t i</sub>	number of tube passes of heat exchanger in branch <i>i</i>
N. :	number of tubes of heat exchanger in branch <i>i</i>
P	atmospheric pressure
$P_{\mathcal{L}}$	power of the fan
Р.	power of the pump
. р Р	water vapor pressure
$\Omega$	heat load in exchanger i
∑i rf	rating factor
rf <sup>°</sup>	rating factor at design conditions
л Т.	ambient temperature
I <sub>amb</sub> T	temperature of the water stream at the tower wit
$I_e$	temperature of the water stream at the lower exit
	temperature of the water stream after make-up
	temperature of the water stream at the lower entrance
I <sub>0,i</sub>	temperature of the water stream that leaves exchanger $E = i$
$T_{wb}$	wet buib temperature
$v_i$	flow velocity in branch i
Vline	flow velocity in the common line
$v_{t,i}$	flow velocity in the tube-side of heat exchanger in $E - i$
Wair	air flowrate through the tower
$W_e$	total water flowrate in the common line
$W_e^{\prime o}$	water flowrate through the tower at design conditions
w'.	water flowrate after forced removal

- $w_{entr}$  water flowrate lost by entrainment
- $w_{evap}$  water flowrate lost by evaporation  $w_i$  water flowrate in branch *i*
- $w_i$  water flowrate in branch *i*  $w_{init}$  make-up water flowrate
- $w_{mu}$  make-up water flowrat  $w_{purge}$  purged water flowrate
- $w_{rem}$  forced withdrawal water flowrate

#### Greek letters

- $\Delta P_i$  total pressure drop in branch *i*
- $\Delta P_{l,i}$  pressure drop in the line of branch *i*
- $\Delta P_{line}$  pressure drop in common line
- $\Delta P_{t,i}$  pressure drop in heat exchanger of branch *i*
- $\Delta P_{total}$  total pressure drop in the system
- $\eta_p$  pump efficiency
- $\lambda_{w}^{r}$  specific enthalpy of the air at the saturation condition
- $\mu_i$  viscosity of cooling water in branch *i*
- $\mu_{line}$  viscosity of cooling water in common line
- $\rho$  density of cooling water
- $\rho_{air}$  density of air

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