THERMODYNAMIC STUDY OF THE NUMBER AND POSITIONING OF THE FEED PUMPS IN THE FEED TRAIN OF A REGENERATIVE STEAM CYCLE

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In this paper an approximate method of analysis is developed to compare the performance of single-pump and split-pump schemes in the non-reheat regenerative steam cycle. The analytical result is shown to agree closely with figures obtained by detailed calculations based on the same standards of comparison of the two schemes as those used in the approximate analysis, although on an alternative basis of comparison the calculated difference between the performances of the two schemes is smaller. The study reveals, however, that on either basis of comparison the efficiency of the cycle is slightly greater for the split-pump arrangement. Detailed calculations for a typical reheat cycle show the same tendency. It is concluded that, on the score of both thermal efficiency and capital cost, the split-pump arrangement is to be preferred.

INTRODUCTION

FROM TIME TO TIME discussion arises as to the optimum number and positioning of feed pumps in the feed train of a regenerative steam cycle, but the problem does not appear to have been treated hitherto on a satisfactory theoretical basis. Salisbury (1950)† applied approximate methods of calculation to assess the effect of altering the position of the boiler feed pump, but neglected the effect of the change in specific volume of the feed water with change in temperature. Kennedy and Hutchinson (1956) treated the problem in general terms by a method which appeared open to question on theoretical grounds. The present paper makes an approximate thermodynamic analysis of the relative performance of non-reheat systems employing the single-pump and split-pump arrangements, and reports on the results of more detailed calculations for both non-reheat and reheat cycles.

Notation

$b = (h - T_c s)$	Steady flow availability function.
h	Enthalpy.
J	Mechanical equivalent of heat.
m	Mass of bled steam supplied to a heater per unit mass of feed water.
$M = \Sigma m$	Total mass of bled steam.
n	Number of heaters.
p	Pressure.
Q_{in}	Heat supplied in the boiler.
Q_{out}	Heat rejected in the condenser.
r	Enthalpy rise of feed water in a heater.
s	Entropy.
T_c	Absolute temperature in condenser.
t	Difference between inlet enthalpy of bled steam and saturation enthalpy of the condensed bled steam in a heater.
$t' = (t+\rho)$	Difference between inlet enthalpy of bled steam and enthalpy of exit drain water in a heater.
v	Specific volume.
W_{net}	Net internal work.
W_0	Work sent out from station.
W_P	Work input to pump motors.
W_T	Internal work output of turbine.
W_W	Work input to feed water = enthalpy rise of feed water in the feed pumps.
$\alpha = v_1/v_2$	
$\beta = \left(1-\dfrac{r}{t'}\right)^{n-1}$	
η_G	Combined efficiency factor for turbine external losses and generator losses.
$\eta_i = W_{net}/Q_{in}$	Internal thermal efficiency of plant.
η_M	Product of pump motor efficiency and mechanical efficiency of pump.
η_O	Overall thermal efficiency of plant.
η_P	Internal isentropic efficiency of pump.
ρ	Reduction in enthalpy of exit drain water below its saturation value at the pressure prevailing in a heater.

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† *An alphabetical list of references is given in the Appendix.*

Suffixes

F Feed water at boiler inlet.

S Steam at boiler exit.

THERMAL EFFICIENCY AND NET WORK

The thermal efficiency of a steam cycle is defined as the ratio of the net work output to the heat supplied. If only the internal isentropic efficiencies of the turbine and feed pump or pumps are considered and stray heat losses are neglected, the net work is given by:

$$\frac{W_{\text{net}}}{J} = (\Delta h_{\text{T}} - \Delta h_{\text{P}}) = (Q_{\text{in}} - Q_{\text{out}})$$

where Δh_{T} is the enthalpy drop of the steam in passage through the turbine, and is equal to the internal work delivered by the steam to the turbine shaft, while Δh_{P} is the enthalpy rise of the feed water in passage through the feed pump(s), and is equal to the internal work delivered to the water by the pump impeller.

These enthalpy quantities refer strictly to stagnation enthalpies, but this distinction will be ignored in the present work. All the above quantities are expressed in terms of unit mass flow of steam to the turbine. The thermal efficiency resulting from the use of the net work output defined above will be described as the internal thermal efficiency, η_i. Account will later be taken of the external losses in the turbine and pumps, and of the generator and pump motor efficiencies. The thermal efficiency expressed in terms of the net work sent out will be described as the overall thermal efficiency, η_0.

The Non-regenerative Cycle

The enthalpy rise of the feed water in passage through the feed pump is given by:

$$\Delta h_{\text{P}} = \frac{\int v dp}{J\eta_{\text{P}}} \doteqdot \frac{v_m \Delta p}{J\eta_{\text{P}}}$$

where v_m is the mean specific volume of the water and Δp the pressure rise in the pump.

Since the specific volume increases with temperature, the work input to the pump is least, and the thermal efficiency is greatest, when the water is compressed at the lowest possible temperature; that is, at the condenser, before any heat is added.

The Completely Reversible Regenerative Cycle

For a completely reversible regenerative cycle with an infinite number of heaters the internal thermal efficiency is given by:

$$\eta_i = \frac{W_{\text{net}}}{Q_{\text{in}}} = \frac{b_{\text{S}} - b_{\text{F}}}{h_{\text{S}} - h_{\text{F}}}$$

It should be noted that the subscript F refers to the state of the feed water at the true final feed temperature, namely, that at boiler inlet; this is not the same as the temperature at exit from the last heater if there is a pump between the last heater and the boiler. Thus, for a given final feed temperature, the internal thermal efficiency of the plant is independent of the arrangement of the feed pump or pumps in the system, so long as this arrangement is compatible with complete reversibility for all processes in the plant; the respective internal work quantities for the turbine and pumps may vary slightly according to the pump arrangement, but their difference will always be the same. A discussion of the criteria for complete reversibility of all processes is outside the scope of the present paper. In any case, reversible cycles are only of academic interest.

The Irreversible Regenerative Cycle with a Finite Number of Heaters

For a reason which will shortly become apparent, there is a thermodynamic advantage in putting the compression work into the feed water at a higher point in the feed train. Against this advantage must be offset the effect of the increase in the work of compression arising from the increased specific volume of the water at the higher temperature. On the relative magnitudes of the effects of these two opposing tendencies depends the thermodynamic superiority or inferiority of the split-pump scheme over the single-pump scheme in a feed system with a finite number of heaters.

Single-pump and Split-pump Arrangements in the Non-reheat Cycle

An approximate analysis is first made for a non-reheat cycle with n surface feed heaters with the drains cascaded, and the alternative pump arrangements shown in Fig. 1 are compared. These are:

System A. Single-pump Scheme. A single pump (pump No. 1) combines the duties of condenser condensate extraction pump and boiler feed pump.

System B. Split-pump Scheme. A condensate extraction pump (pump No. 1) delivers to a pressure sufficient to prevent ebullition at the suction of the main boiler feed pump (pump No. 2), which is placed after the nth heater.

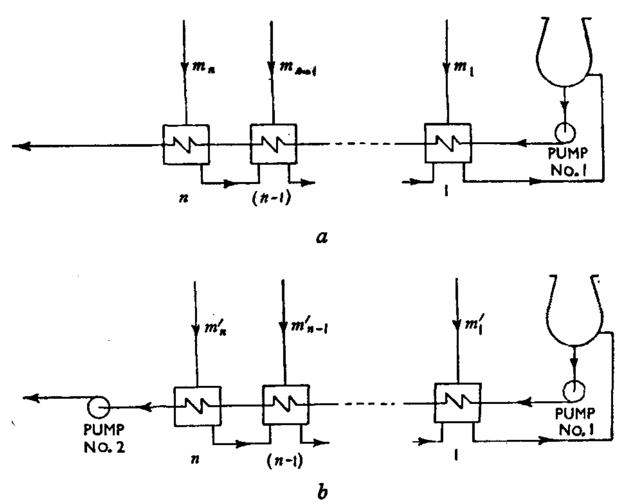


Fig. 1. Feed Heating Systems, Non-reheat Cycle

These pump arrangements have been chosen for the sake of simplicity and it is not suggested that they would be adopted in practice, but the method of calculation can readily be applied to more practical systems.

For the purpose of analysis certain simplifying assumptions are made and their effects considered later. To overcome, though not to neglect, the complication resulting from the variation in the enthalpy of water with varying pressure at a given temperature, it is assumed that the exit enthalpy (and not the exit temperature) of the feed water leaving each heater but the nth is the same in both systems A and B. It is also assumed that the bled steam condition at the inlet to each heater is the same in the two cases; the heater terminal temperature difference is consequently not quite the same in the two cases, and the effect of this is discussed later.

With the same final feed temperature at boiler inlet, the principal effect of changing the arrangement from A to B is then a decrease in enthalpy rise in the nth heater, as a result of the introduction of pump No. 2 between this heater and the boiler, and an increase in enthalpy rise in the first heater as a result of the reduction in enthalpy rise in pump No. 1. Per unit mass flow to the turbine, this brings about a relatively large reduction in the steam quantity bled to the nth heater, and consequential small increases in the steam quantities bled to all the other heaters as a result of the reduced drain quantity passing through them, whilst there is a further increase in the steam quantity bled to the first heater as a result of the increased enthalpy rise of the feed water in this heater.

Since the higher the pressure at which steam is bled the greater is the reduction in turbine work output due to bleeding, a given reduction in steam quantity bled to the nth heater results in a greater increase in turbine work output than results from a similar reduction in bled steam quantity to a lower heater. Thus the thermodynamic advantage of putting the pump work into the feed water at a higher point in the feed train is clearly seen. It remains to be seen, however, to what extent this increase in turbine work output is offset by the increased pump work input resulting from the increased specific volume of the feed water at the higher temperature. If there is an increase in the total quantity of steam bled to all heaters as a result of the above changes for the individual heaters, then the steam flow rate to the condenser will be reduced, and so the heat quantity rejected in the condenser will also be reduced; the net work, W_{net}, and the internal thermal efficiency will thus be increased in spite of an increase in total pump work input. The effect can be examined analytically by using approximate methods described in the author's paper on the regenerative cycle (Haywood 1949).

APPROXIMATE ANALYSIS

The method is based on the fact that the difference, t, between the enthalpy of the bled steam and the saturation enthalpy of the condensed bled steam, is very nearly the same for all heaters, and in the analysis is assumed to be the same. It was shown in the author's paper that the optimum condition for maximum efficiency occurs, to a first approximation, when the enthalpy rise, r, of the feed water in a heater is the same for all heaters. Consequently if, with surface heaters with the drains cascaded, all the heaters have approximately the same terminal temperature differences, the difference between the enthalpy of the inlet bled steam and the enthalpy of the exit drain water will be very nearly the same for all heaters and will equal $t' = (t+\rho)$ where ρ is the reduction in enthalpy of the exit drain water below its saturation value at the pressure prevailing in the heater. With this data, an energy balance can be drawn up for each heater in turn, starting at the nth, to evaluate the changes in bled steam quantities resulting from the change from system A to system B. With the type of feed train chosen, unit mass flow rate to the turbine corresponds to unit mass flow rate of feed water through all the heaters and, in the analysis, all flow and energy quantities relate to unit mass of steam supplied to the turbine. Work is expressed in thermal units.

Under the above conditions, with a constant final feed temperature at boiler inlet, the reduction in enthalpy rise in the nth heater in system B, when compared with system A, is equal to the enthalpy rise, Δh_2, of the feed water in pump No. 2. An energy balance for the nth heater then shows that the resulting change in steam quantity bled to this heater is given by

$$\delta m_n = -\frac{\Delta h_2}{t'}$$

This reduction in steam quantity bled to the nth heater is reflected in the $(n-1)$th heater by an equal reduction in the quantity of drain water passing through the latter, and an energy balance for the $(n-1)$th heater shows that this results in an increase in the steam quantity bled to this heater of

$$\delta m_{n-1} = +\frac{r}{t'} \cdot \frac{\Delta h_2}{t'}$$

The total change in steam quantity bled to these two heaters is thus given by

$$\delta m_n + \delta m_{n-1} = -\frac{\Delta h_2}{t'}\left(1-\frac{r}{t'}\right)$$

This quantity is the reduction in the quantity of drain water passing through the $(n-2)$th heater, and by repeating the above calculation progressively down to the first heater it is seen that the total change in steam quantity bled to all n heaters, on account of the introduction of pump No. 2 between the nth heater and the boiler, is given by

$$\sum_n^1 \delta m = -\frac{\Delta h_2}{t'}\left(1-\frac{r}{t'}\right)^{n-1} \quad . \quad . \quad (1)$$

The change from system A to system B results in a change, Δh_1, in the enthalpy rise in pump No. 1. Under the conditions cited above, the resultant change in the enthalpy rise of the feed water in the first heater causes a further change in the steam quantity bled to this heater, of magnitude

$$\delta m_1 = -\frac{\Delta h_1}{t'} \quad . \quad . \quad . \quad . \quad (2)$$

where Δh_1 is in fact negative, so that δm_1 is positive.

The total change in steam quantity bled to all the heaters in changing from system A to system B is thus given by adding equations (1) and (2) to give

$$\Delta M = -\left[\frac{\Delta h_1}{t'} + \frac{\Delta h_2}{t'}\left(1-\frac{r}{t'}\right)^{n-1}\right]$$

Per unit mass of steam supplied to the turbine stop valve, the steam flow to the condenser is thus reduced by an amount equal to ΔM and, since the heat rejected in the condenser per unit mass of steam is, to a first approximation, equal to t, and the reduction in heat quantity rejected in the condenser is equal to the increase in net work, ΔW_{net}, there is an increase in net internal work resulting from the change from A to B which is given by

$$\Delta W_{net} = -\frac{t}{t'}\left[\Delta h_1 + \left(1-\frac{r}{t'}\right)^{n-1} \Delta h_2\right] \quad . \quad (3)$$

The factor $\left(1-\frac{r}{t'}\right)^{n-1}$ by which Δh_2 is multiplied gives an inverse measure of the thermodynamic advantage of putting the pump work into the feed water at a higher point in the feed train, in that the smaller this factor the greater is ΔW_{net}.

If the pressure rise in pump No. 2 is Δp_2, and if both pumps have an internal isentropic efficiency of η_p, then Δh_1 and Δh_2 are given by

$$\Delta h_1 = -\frac{v_1 \Delta p_2}{\eta_p J} \text{ and } \Delta h_2 = \frac{v_2 \Delta p_2}{\eta_p J} \quad . \quad . \quad (4)$$

where v_1 and v_2 are the mean specific volumes of the feed water passing through the respective pumps. Per unit mass of steam supplied to the turbine stop valve, the increase in net work in changing from system A to system B is thus

$$\Delta W_{net} = \left[\frac{v_1}{v_2} - \left(1-\frac{r}{t'}\right)^{n-1}\right]\frac{t}{t'} \cdot \Delta h_2$$

$$= (\alpha - \beta)\frac{t}{t'} \cdot \Delta h_2 \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Whether the net work is greater or less in scheme B than in scheme A depends on the relative magnitudes of α and β. If β is less than α the thermodynamic advantage of putting the compression work into the feed water at a higher point in the feed train will more than compensate for the resulting increase in compression work.

The Relative Magnitudes of α and β

Both α and β are principally dependent on the final feed temperature, and are little affected by variation of the operating pressure. A fairly close estimate of the quantities for typical conditions may therefore be made in terms of the final feed temperature. v_1 and v_2 may be taken as the saturation specific volumes at condenser temperature and final feed temperature respectively, and the value of α so calculated is shown in Fig. 2 for a vacuum of 29 inches mercury (barometer, 30 inches). If, for the purpose of this approximate estimate, the feed water enthalpy at inlet to the first heater is assumed to be 50 B.Th.U. per lb., and r is taken to be 70 B.Th.U. per lb., then the final feed temperature corresponding to any chosen number of heaters can be determined approximately, and β can be calculated in each case if a value of t' is assumed. The points on the dotted curve in Fig. 2 show such calculated values of β for 4, 5, 6, and 7 heaters respectively, and a value of $t' = 1{,}000$ B.Th.U. per lb.

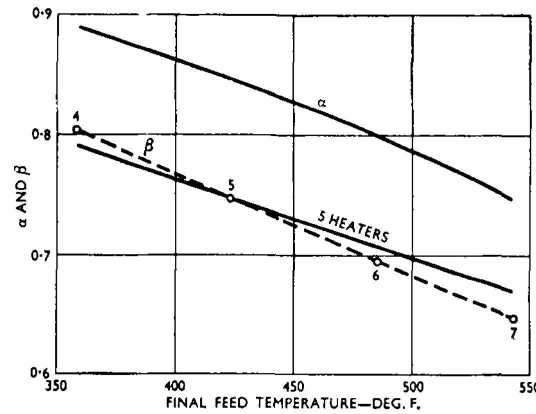


Fig. 2. α and β against Final Feed Temperature

The numbers on the curve for β relate to the number of heaters.

The two curves for α and β are closely parallel to each other over the entire range of feed temperature shown, α being a practically constant small amount greater than β, so that ΔW_{net} in equation (5) will always be positive in this range. This means that the net internal work output of the split-pump scheme will be greater than that of the single-pump scheme for a given heat input.

That β is relatively insensitive to variations of r and t' is seen by calculating the value of β for the same total enthalpy ranges as before, but keeping the number of heaters constant at five over the entire range, so that r varies. With suitable allowance for the consequent variation of t', the calculated value of β is shown as the full curve in Fig. 2. The difference between α and β is still retained, though slightly reduced at the higher temperatures. However, it should be pointed out that the dotted curve is more likely to represent practical conditions, since the higher the final feed temperature for which the plant is designed the greater will be the number of heaters.

Effect on the Net Work Sent Out from the Station

W_{net} is the difference between the internal work output of the turbine and the work quantity put into the feed water in the pumps, so that in the above evaluation of ΔW_{net} no account has been taken of the effect of turbine external losses and of the efficiencies of the generator and pump motor, nor of the mechanical efficiency of the pump. This effect must be evaluated before the overall effect on the work sent out from the station can be assessed. The following relations hold

$$W_{net} = W_T - W_W \quad . \quad . \quad . \quad (6)$$

$$W_W = \eta_M W_P \quad . \quad . \quad . \quad (7)$$

$$(W_0 + W_P) = \eta_G W_T \quad . \quad . \quad . \quad (8)$$

It can readily be shown from equations (6), (7), and (8) that the change ΔW_0 in work sent out is related to ΔW_{net}, as already evaluated, by the equation

$$\Delta W_0 = \eta_G \Delta W_{net} - \left[\frac{1}{\eta_M} - \eta_G\right] \Delta W_W \quad . \quad (9)$$

where

$$\Delta W_W = (\Delta h_1 + \Delta h_2) = \Delta h_2 \left[1 - \frac{v_1}{v_2}\right] \quad . \quad (10)$$

Equations (4), (5), (9), and (10) give all the information necessary for the overall station efficiencies for schemes A and B to be compared, since they enable the difference between the work sent out from the station in the two cases to be determined. The equations are best used as they stand, but they can be combined into the single equation given below for ΔW_0 in terms of Δp_2

$$\Delta W_0 = \left[\eta_G \frac{t}{t'}(\alpha - \beta) - \left(\frac{1}{\eta_M} - \eta_G\right)(1-\alpha)\right] \frac{v_2 \Delta p_2}{\eta_P J} \quad . \quad . \quad (11)$$

EVALUATION BY APPROXIMATE ANALYSIS, AND BY DETAILED CALCULATION, FOR SPECIFIC CONDITIONS

To relate the analysis to a specific case, and to check by detailed calculation the answer so obtained, the following conditions were chosen:

Steam conditions at turbine stop valve	1,500 lb. per sq. in. abs./ 1,000 deg. F.
Condenser vacuum .	29 inches of mercury (barometer, 30 inches)
Final feed temperature (at boiler inlet) . .	450 deg. F.
Number of heaters . .	5
Intermediate feed pressure in system B . .	600 lb. per sq. in. abs.
Internal isentropic efficiency of pumps . .	75 per cent

For these conditions a turbine expansion line was assumed, and detailed calculations were made for systems A and B for the conditions laid down as the basis for the foregoing analysis. For the purpose of this comparison the final feed pressure was taken to be equal to 1,500 lb. per sq. in. abs. The bled steam pressures in system A were chosen to give approximately equal enthalpy rises in all heaters and heater terminal temperature differences of 10 deg. F. at feed water outlet. The temperature difference between the exit drain water and the inlet feed water was taken as 5 deg. F. for all heaters. The bled steam condition, the exit enthalpy of the drain water from each heater, and the exit enthalpy of the feed water from each heater but the last, were kept the same in system B as in system A. The results of these detailed stage-by-stage calculations are summarized in Table 1, where the value of ΔW_{net} so determined is compared with the value calculated from equation (5).

Table 1. Results of Calculations

		System A	System B	Change from A to B
Enthalpy rise of feed water in pump No. 1, B.Th.U. per lb.		5·935	2·376	$\Delta h_1 = -3{\cdot}559$
Enthalpy rise of feed water in pump No. 2, B.Th.U. per lb.		—	4·284	$\Delta h_2 = +4{\cdot}284$
Total work of compression, B.Th.U. per lb. . .		5·935	6·660	$\Delta W_W = +0{\cdot}725$
ΔW_{net}	From detailed calculations . .	—	—	+0·399
	From equation (5) .	—	—	+0·389

The agreement between the values of ΔW_{net} obtained by detailed calculation and from equation (5) is seen to be good. Of particular interest is the fact that, in spite of the negative work in the cycle being greater in system B than in system A, the net work and therefore the thermal efficiency of B is greater than that of A. The reason for this was seen in the discussion of equation (5) and Fig. 2. It is further illustrated by showing the calculation of ΔW_{net} from equation (3), although equation (5) would usually be used. Using the mean values for r, t, and t' from the detailed calculations (in the absence of detailed calculations these can be estimated readily to sufficient accuracy) equation (3) gives

$$\Delta W_{net} = -\frac{t}{t'}\left[\Delta h_1 + \left(1 - \frac{r}{t'}\right)^{n-1} \Delta h_2\right]$$

$$= \frac{930}{1{,}012}\left[3{\cdot}559 - \left(1 - \frac{75{\cdot}5}{1{,}012}\right)^4 \times 4{\cdot}284\right]$$

$$= \frac{930}{1{,}012}(3{\cdot}559 - 0{\cdot}7332 \times 4{\cdot}284)$$

$$= \frac{930}{1{,}012}(3{\cdot}559 - 3{\cdot}141) = 0{\cdot}384$$

Thus although Δh_2 is greater in magnitude than Δh_1, the factor $\left(1 - \frac{r}{t'}\right)^{n-1}$ by which Δh_2 is multiplied results in ΔW_{net} being greater for system B than for system A: that is, the thermodynamic advantage of putting the work into the feed water at a higher point in the feed train more than offsets the effect of the increase in pumping work arising from the greater specific volume of the water at the higher temperature. It may be noted that, for the conditions chosen, the magnitudes of α and β are respectively 0·8320 and 0·7332: these are in close agreement with the approximate values read from Fig. 2 at a final feed temperature of 450 deg. F.

In terms of the net work sent out from the station, W_0 is greater for system B than for system A by an amount ΔW_0, given by equation (9). Using the value of ΔW_{net} obtained by detailed calculation, and taking $\eta_G = 96$ per cent and $\eta_M = 93$ per cent, this gives

$$\begin{aligned}\Delta W_0 &= 0{\cdot}96\times0{\cdot}399-0{\cdot}1153\times0{\cdot}725 = 0{\cdot}383-0{\cdot}084\\ &= 0{\cdot}299 \text{ B.Th.U.}\end{aligned}$$

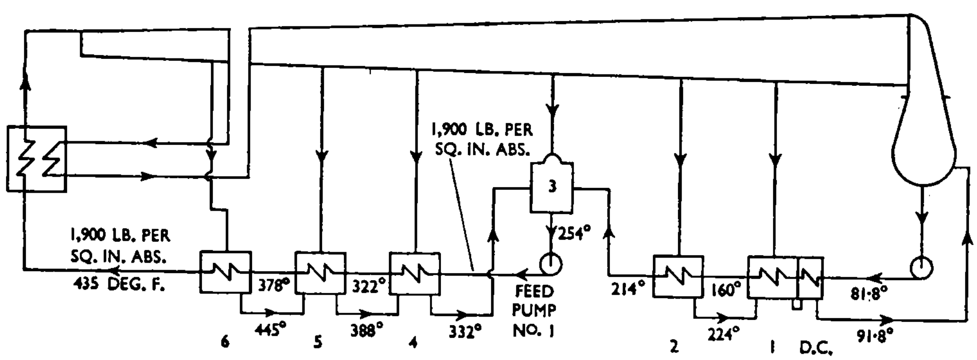
Thus, in spite of an increase of pumping power for system B of 0·725 B.Th.U., or about 0·17 per cent of the work sent out, there is an *increase* of about 0·07 per cent in the work sent out. These quantities are small but in the present context it is the effect that is significant, rather than its magnitude.

An Alternative Basis of Comparison

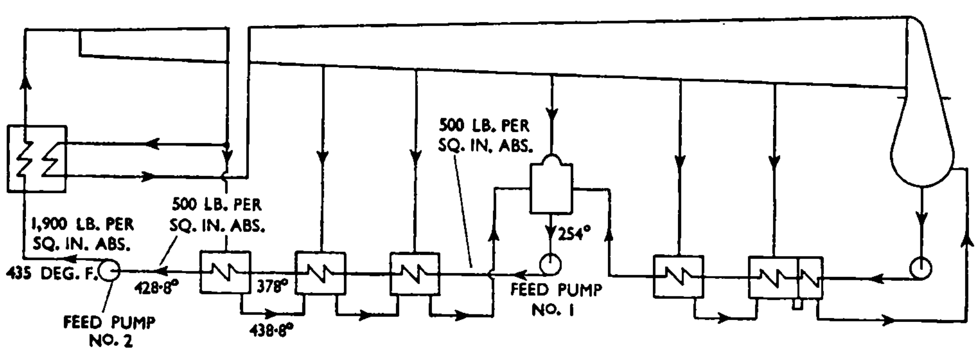
It might be questioned whether the basis chosen above for the comparison of the two systems, which has been devised in order to render an analytical solution possible, is the best that can be made. Ideally, the optimum conditions for both systems should be determined individually, and then the performances of the two systems at their respective optimum conditions compared. This would be far too tedious and is not justified by the nature of the problem. In practice, the two systems would probably be designed for the same heater terminal temperature differences, rather than for the same feed water and drain enthalpies at exit from each heater but the last. A careful detailed calculation was therefore made on this alternative basis, keeping the bled steam conditions the same for all heaters but the last; the bled steam pressure to the last heater was reduced for system B in order to keep the difference between the bled steam saturation temperature and the feed water outlet temperature unaltered.

On this alternative basis of comparison, system B still shows a gain over system A, but the increase in W_{net} is reduced from 0·399 B.Th.U. (0·09 per cent), to 0·117 B.Th.U. (0·03 per cent), while ΔW_0 is reduced from 0·299 to 0·029 B.Th.U. That the original basis of comparison favours B more than does the new basis is explained by the fact that on the former basis the last heater, which uses the highest grade steam, carries all the reduction in feed water enthalpy rise due to the introduction of pump No. 2, whereas on the new basis all the heaters are affected by the change in enthalpy of the feed water with pressure. It is interesting to note, however, that on either basis the split-pump scheme shows a gain over the single-pump scheme, in spite of the smaller pumping power in the latter.

As a point of academic interest only, it might be noted that detailed calculation shows that an arrangement having individual pumps between each heater gives the best performance of all, although the margin is small and the pumping power is greater than in the other two arrangements.



a Single-feed-pump scheme.



b Split-feed-pump scheme.

Fig. 3. Feed Heating Systems for Reheat Cycle

THE REHEAT REGENERATIVE CYCLE

When reheating is used, the same two opposing factors as are experienced in the non-reheat cycle influence the change in efficiency resulting from a change in pump position. It is not profitable to attempt to apply an approximate method of calculation to the reheat cycle, and an exact determination of the effect of changing the pump position must be made by very careful detailed calculation. Such a detailed calculation has been made in collaboration with Mr. B. Wood* for the typical practical conditions illustrated in Fig. 3. The calculations related to a plant operating with initial steam conditions of 1,500 lb. per sq. in. abs./1,000 deg. F., reheating to 1,000 deg. F., and a vacuum of 28·9 inches mercury (barometer, 30 inches).

For the split-pump scheme of Fig. 3*b* the reheat tapping point coincided with the highest heater tapping point. In order that the two calculations should be on a comparable basis the reheat pressure was kept the same for the single-pump scheme of Fig. 3*a* although this involved an impracticable position for the reheat point, in that it was then just above the highest heater tapping point. The final feed pressure was taken as 1,900 lb. per sq. in. abs. in both cases, with an intermediate feed pressure of 500 lb. per sq. in. abs. for the split-pump scheme. In both cases the calculations were based on a terminal temperature difference of 10 deg. F. between the bled steam saturation temperature and the feed water outlet temperature in all heaters, and the exit drain water temperature was taken as being equal to the bled steam saturation temperature. It is of interest that the split-pump scheme still showed a small gain over the single-pump scheme although the difference was so small as to be negligible for all practical purposes, being 0·02 per cent on net work and less than 0·01 per cent on work sent out.

CONCLUSIONS

The paper has given an analytical and detailed comparison of the performance of single-pump and split-pump schemes, and has explained why the latter gives the slightly higher thermal efficiency. Although this advantage of the split-pump scheme is so small as to be practically negligible, it is reinforced by other advantages of a practical and economic nature. A practical advantage is the fact that, with the split-pump arrangement, not all the heaters are subjected to full boiler pressure. A point in favour of the single-pump scheme is the provision of only one pump, with an input smaller than the combined inputs of the two pumps in the split-pump scheme. However, Kennedy and Hutchinson (1956) have estimated that for 120-MW. units, although the single pump and motor might cost some 30 per cent less than the two pumps and motors in the split-pump arrangement, the cost of the heaters might be 50 per cent higher when using a single pump, on account of the fact that the heaters are all subjected to full boiler pressure; they consequently estimated that the total expenditure for a 120-MW. unit would be at least £30,000 higher for the single-pump scheme. When there is added to this the continuous small saving on fuel, the split-pump scheme is seen to be preferred on both counts.

Throughout the paper reference has been made to the true final feed temperature at boiler inlet. Only in terms of this temperature is the efficiency of a completely reversible cycle the same for all possible alternative arrangements which involve no irreversibilities. In spite of the fact that this is the only true final feed temperature, it has been customary, for obvious practical reasons, for makers' guarantees to be given in terms of the feed temperature at exit from the last heater. If this temperature were kept the same for systems A and B, then the advantage would be still more in favour of the split-pump scheme, because its true final feed temperature is greater than that of the single-pump scheme by an amount equal to the temperature rise in pump No. 2, which is of the order of 4 deg. F. for the conditions quoted. The additional gain in thermal efficiency consequent on the higher mean temperature of heat reception is quite artificial, however, since it results from a basis of comparison which is unscientific.

* *Messrs. Merz and McLellan.*

APPENDIX

REFERENCES

HAYWOOD, R. W. 1949 Proc. I.Mech.E., vol. 161, p. 157, 'A Generalized Analysis of the Regenerative Steam Cycle for a Finite Number of Heaters'.

KENNEDY, G. F., and HUTCHINSON, F. J. 1956 Proc. I.Mech.E., vol. 170, p. 341, 'Power Station Auxiliary Plant'.

SALISBURY, J. K. 1950 'Steam Turbines and Their Cycles', p. 528 (John Wiley and Sons, Inc., New York; Chapman and Hall, London).

Communications

Mr. F. J. Barclay, B.Sc. (*Associate Member*), A.M.I.E.E., wrote that he would like first to ask the author to state precisely why he found the method of Kennedy and Hutchinson open to question on theoretical grounds.

The reasoning in the paragraph headed 'The Non-regenerative Cycle' would have been complete if he had added that as the work input to the feed pump was increased the heat input to the cycle was reduced for a constant work output from the turbine.

In the paragraph headed 'Approximate Analysis' the equal enthalpy rise optimization applied only to the uncommon system of all cascaded drains. It would not apply to the (even less common) system of having a drains pump at every heater.

It was difficult to obtain any satisfaction from the approximate method since the physical picture was obscured. Moreover, there seemed to be no good reason why the type of cycle (reheat or non-reheat) should greatly affect the issue between the pumping schemes.

A complete picture was obtainable (tediously) by taking the precision heat balances (unfortunately not published) prepared by the author and Mr. Wood and converting them into equally precise entropy balances (Keller 1950)* which would show exactly which internal losses were varying. Once having obtained that picture a short-cut method would be obtained since the relevant losses or irreversibilities could be calculated individually and summed.

That was indeed the net effect of the Kennedy and Hutchinson paper, although they had guessed rather than established by an analytical procedure which were the relevant things to calculate.

Mr. C. A. Meyer (Philadelphia, Pa.) wrote that the analysis appeared to be very good but somewhat academic. The author had used available energy methods to evaluate the thermodynamic effects. However, many practical items such as the effect of splitting the pump on pump efficiency, etc., were presented. As the author had stated in the conclusion, 'This advantage is so small as to be practically negligible', there might be some question as to the purpose of the paper. It might be concluded that it was for the purpose of showing that economic factors were more important than thermodynamic ones.

Mr. B. Wood, M.A. (*Associate Member*), wrote that it was interesting to see that the author's method, which depended on the assumption of a condition line giving a constant difference between enthalpy of steam and enthalpy of water at the extraction point, led to very much the same sort of answer as the more practical calculations done by the method he had been using for some twenty years. His experience had confirmed that with polytropic expansion at 85 per cent stage efficiency that difference was not very far from constant in non-reheat cycles particularly up to moderate pressures and superheats.

He noted that with both the author's method and his own the calculations showed a slight advantage in favour of split pumps. He would, however, point out that that advantage applied only to the somewhat idealized assumptions. In practice it was likely that the single feed pump scheme would, in fact, be more economic, since when using pumps of normal design with practical gland losses the split pump system suffered from having to pump the gland leakage losses of the additional intermediate pressure glands and the further augmented leakage from both glands of the high-pressure pump which arose from their operating with water of higher temperature. Clearances in hot glands ordinarily had to be larger, though if an independent cold-water-supply was provided for gland cooling that might not apply. Thus the quantity of water pumped in the split pump system was likely to be, say, 10 per cent higher than in the single pump system and that was just about sufficient to turn the scale. He regretted that he had not pointed that out to the author originally. That was the sort of thing which tended to be forgotten when following too academic an approach or attempting to be completely judicial in comparisons. With higher pressures than the 1,500 lb. per sq. in. considered that effect tended to become even more serious, but the pump might then have to be divided for other reasons whether the two parts of the pump were separated in temperature or not. It thus became necessary to settle in each case how high the gland losses were likely to be and it was impossible to generalize.

* *Keller, Allan 1950 Trans. A.S.M.E., vol. 72, p. 949, 'The Evaluation of Steam Power-plant Losses by Means of the Entropy-Balance Diagram'.*

Author's Reply

Mr. R. W. Haywood, in reply to the communications, wrote that Mr. Barclay's contribution had stimulated him to further thought, although he was afraid that he found himself in disagreement with much that Mr. Barclay had written. He had not thought it necessary to carry to completion the reasoning in the paragraph headed 'The Non-regenerative Cycle', as the result appeared to him sufficiently obvious. If the reasoning were carried to completion, however, Mr. Barclay's version appeared to him to be inadequate. It was necessary to show that the decrease in η_i due to decrease in W_{net} was greater than the increase in η_i due to decrease in Q_{in}. That was established by writing, from the definition of η_i,

$$\frac{\Delta\eta_i}{\eta_i} = \frac{\Delta W_{net}}{W_{net}} - \frac{\Delta Q_{in}}{Q_{in}}$$

Since $\Delta W_{net} = -\Delta W_w = \Delta Q_{in}$, where ΔW_w was the increase in work input to the feed water per unit mass of fluid and, since W_{net} was always less than Q_{in}, it followed that $\Delta\eta_i$ was negative for an increase in pump work. Alternatively that could be seen by using the above relations to obtain the expression

$$\frac{\Delta\eta_i}{\eta_i} = -\frac{\Delta W_w}{W_{net}}(1-\eta_i)$$

He disagreed with Mr. Barclay's statement that, in the paragraph headed 'Approximate Analysis', the equal enthalpy rise optimization applied only to a system in which all drains were cascaded. Mr. Barclay would find by reference to Haywood (1949) that the equal enthalpy rise optimization was actually developed for a system employing direct-contact heaters and drain pumps, and not for one employing surface heaters with the drains cascaded. He had, nevertheless, pointed out in that paper that this optimization applied, to a first approximation, to the latter type of system as well, and therefore also to a system incorporating both direct-contact and surface heaters. The degree of approximation was dependent on the simplifying assumptions which were necessary to allow of an analytical solution; those were listed in his paper.

He again found himself in disagreement with Mr. Barclay over his criticism that the approximate method obscured the physical picture, because he thought it brought out clearly, although not to a high degree of accuracy, the two separate effects arising from a change in pump position: that due to the increase in specific volume of water with temperature, and the compensating effect arising when feed heating was employed. The treatment of Kennedy and Hutchinson (1956) completely ignored the latter effect, although they might have thought that they had allowed for it in their so-called 'efficiency at which power was regained'. It was only if that effect was understood that it could be seen why the type of cycle (reheat or non-reheat) could affect the result; he had not said it *greatly* affected the issue. Although he agreed that Keller's method of 'entropy balances' had its uses, he did not think it would give any clearer *physical* explanation of the factors involved than did his own method. Considerations of space had led him not to include the energy balances for his detailed calculations, as he had not thought their inclusion would add sensibly to the value of the paper.

In his own contribution to the discussion on Kennedy and Hutchinson (1956) he had asked if the authors could give a rigorous justification of their method of calculation. Unfortunately they had not done so in their reply, and his belief that it would not be possible to defend their method rigorously had been strengthened by Mr. Barclay's statement that they had guessed, rather than established by an analytical procedure, which were the relevant things to calculate. He had already pointed above to one of the defects in Kennedy and Hutchinson's treatment; similar criticism had been made by Mr. Thomas in his contribution to the discussion on their paper. Apart from that, their Table 6 appeared to him to be subject to some confusion of ideas, and the precise basis of comparison to which it related was far from clear. Consequently he did not think it worth while to discuss it piecemeal in the absence of a fuller and more rigorous statement of the basis of calculation, particularly as, if he did so, it would add very appreciably to the length of his reply to the discussion on his own paper. That their method showed no distinction between the cases when there was feed heating and when there was no feed heating was indication that there was something basically wrong with their treatment, for they had themselves drawn attention to the fact that alteration of the pump arrangement did not affect the net internal work and internal thermal efficiency so long as the regenerative cycle was completely reversible (implying, among other things, an infinite number of heaters); whereas, in the absence of feed heating, the net internal work and internal thermal efficiency were affected by an alteration in pump arrangement because there was no compensating alteration in turbine work output. When there was feed heating, such an alteration was reflected in a compensating change in turbine work output as a result of changes in the bled steam quantities. His own method showed the origin of that compensating effect and related it to the degree of irreversibility in the feed-heating system by taking account of the number of heaters.

He made no claim that his analytical method gave an exact answer to the problem; it was not designed to do so, but to bring out the physical factors involved in a change in pump position when there was feed heating. Indeed, he had drawn attention to the fact that the theoretical basis gave a more optimistic assessment of the split-pump scheme than did a more realistic and practical basis, and furthermore that neither of those gave the ideal solution, which should be obtained by determining optimum conditions for both schemes individually. To determine the true optimum division of enthalpy rises amongst the heaters in each scheme would be a formidable task, but if it were done there was no doubt that it would alter slightly the comparison between the two schemes.

That led him to recall that alteration in pump arrangement made no difference to W_{net} so long as the regenerative system was completely reversible in all cases, whereas with no feed-heating increase in W_W led to a decrease in W_{net} for a given Q_{in}. Thus, in terms of W_{net}, there was no difference between single- and split-pump arrangements when the feed-heating system was fully reversible, while the advantage lay with the single-pump scheme when there was no feed heating. Since the degree of irreversibility in the feed-heating system decreased with increase in the number of heaters, it was possible that with a large number of heaters (and in that context five was quite a large number) the advantage would still lie very slightly in favour of the single-pump arrangement if both schemes were properly optimized. On the other hand, his own detailed calculations on the alternative basis of comparison, which was a basis such as might be expected in practice where detailed optimization of the two arrangements was not feasible, showed a slight advantage in favour of the split-pump arrangement in that it gave a slightly larger W_{net}. Consequently he thought that, to the degree of accuracy in which he was interested, the practical engineer would be content to accept the dictum that thermodynamically (that is, in terms of W_{net} for a given Q_{in}), there was little or nothing to choose between the two systems. (It might be noted from equation (6) that if W_{net} were the same for both, W_T would be higher in the split-pump scheme than in the single-pump scheme because W_W was higher, so that the turbine heat rate would be lower in the split-pump scheme. In that respect the turbine heat rate might be misleading, and it must not be used to compare overall station performance.)

Mr. Meyer had concluded very rightly that the paper had an academic purpose in clearing up the thermodynamic considerations involved. Having done that, attention could be concentrated on the economic and other factors, which might influence a choice between the two systems. One such factor had been mentioned by Mr. Meyer and another by Mr. Wood. Others had been cited by several contributors to the discussion on Kennedy and Hutchinson (1956), but discussion of those was outside the scope of the present paper.

Having concluded that the difference in W_{net}, namely, the difference in net internal work per unit mass flow between the two schemes, was small enough to be considered negligible, equation (9) showed that, for a given Q_{in}, the change in work sent out from the station was given by

$$\Delta\overline{W}_0 = -\left[\frac{1}{\eta_M}-\eta_G\right]\Delta\overline{W}_W$$

where the bar signified total work quantity per unit time, as distinct from work quantity per unit mass. On the assumption that Table 6 of Kennedy and Hutchinson (1956) related to a given Q_{in}, the value of $-\Delta\overline{W}_0$, so calculated, could be compared with their 'net reduction in station output'. In their example $\Delta\overline{W}_W$ was $1{,}818-1{,}656 = 162$ kW. Using his own values of 93 per cent for η_M and 96 per cent for η_G, $-\Delta\overline{W}_0$ was about 19 kW., compared with their value of 11 kW. From his equation (6), ΔW_T was equal to ΔW_W if W_{net} was zero, so their 'net increase in turbo-alternator size' could also be compared with $\eta_G\Delta\overline{W}_W$, giving a figure of 156 kW. That compared with their figure of 155 kW., but he regarded the agreement as largely fortuitous. He would not translate those figures into money values because, as Mr. Barclay had pointed out in his contribution to their paper, it was not realistic to give a comparison in terms of the cost of the net increase in turbo-alternator size, since alternators were usually of standard size. However, he did not agree with Mr. Barclay's alternative suggestion, namely, that that item should be replaced by the cost of an outage of the same magnitude. If single-pump and split-pump schemes were to be compared for sets of the same output at the alternator terminals, then the whole basis of comparison was altered from that already given, since that was based on the same total rate of heat supply to the fluid in its passage through the boiler. Nevertheless, a change to the new basis could be made readily: if, per unit mass flow rate, W_A was the alternator output, then

$$W_A = (W_0+W_P) = \eta_G W_T$$

If $\delta\overline{W}_A$ were to be zero to give the same alternator output for both systems, then $\delta\overline{W}_T$ must be zero. (δ there related to the difference between two systems with the same kW. output from the alternator, while Δ related to the difference between two systems with the same heat input in the boiler per unit mass of fluid.) It had been shown that in the split-pump scheme both W_W and W_T were greater than in the single-pump scheme, so that if $\delta\overline{W}_T$ were to be zero the mass flow rate in the split-pump scheme would have to be less than that in the single-pump scheme (that accorded with the fact, already noted, that the *turbine* heat rate was less). If ΔW_T was the difference between the internal work outputs of the turbines per unit mass flow in the two schemes, then the difference between the mass flow rates of feed water to the boiler in the two schemes would be given to a good approximation by

$$\delta m . W_T = -m.\Delta W_T = -\Delta\overline{W}_T$$

Hence the total heat input rate in the boiler would be less in the split-pump scheme by an amount given by the relation

$$\delta\overline{Q}_{in} = Q_{in}.\delta m = -\frac{Q_{in}}{W_T}.\Delta\overline{W}_T \doteqdot -\frac{\Delta\overline{W}_W}{\eta_i}$$

since $\Delta W_T = \Delta W_W$ when ΔW_{net} was zero, and W_T/Q_{in} was approximately equal to η_i.

The effect of that difference in Q_{in} in the two schemes could then be expressed in terms of differences in capital costs and capitalized fuel costs. Against that must be offset the effect of the smaller net station output in the split-pump scheme. It could readily be shown from the above relations that, allowing for the smaller mass flow rate in the split-pump scheme, the net station output in the split-pump scheme was smaller by an amount given by the relation

$$\delta W_0 = -\frac{\Delta\overline{W}_W}{\eta_M}\left(1-\frac{W_W}{W_T}\right)$$

The cost of that decrease in station output could then be capitalized in the usual manner. It would then be possible to compare the final figures arrived at in that way with the difference between the capital costs of the pumps, motors, and heaters in the two schemes.

In dealing with those matters which had arisen from Mr. Barclay's contribution to the discussion, he had been led rather far from his original intentions, but he hoped that he had helped towards a better understanding of the relative performances of the single- and split-pump schemes.