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Modelling the power production of single and multiple extraction steam turbines

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ABSTRACT

A thermodynamic model for the prediction of the operating performance of back pressure steam turbines with single and multiple extractions is presented. The approach takes the advantage of previous models reported in the literature and incorporates some improvements that allow for the prediction of the shaft work under changes in the operating conditions such as steam flow rate and pressure of each of the extractions. Simulation under changed operating conditions is conducted through the use of expressions that correlate the temperature and enthalpy of the steam as a function of the extraction pressure. The model incorporates the calculation of the isentropic efficiencies of each stage. The validity of the model is demonstrated by comparing the results with those of commercial turbines reported in the open literature.

1. Introduction

There are two aspects related to the thermal integration of steams turbines into background processes. One is the targeting of power ahead of design that commonly is carried out using approximate thermodynamic models and the second is the actual selection of the turbine that will do the job of simultaneously producing heat and power to satisfy the needs of the process. In this context, targeting and selection are issues that are closely related. This relation was rightly appreciated by Mavromatis and Kokossis (1998) who developed an approach known as the hardware model for steam turbines. This model is a novel tool for targeting the cogeneration potential in a process, since it is based on typical turbine performance data. However, in the process of reconciling targeting and selection, there is still a need to test the theoretical model against true equipment performance, particularly in sight of the need to predict power production under a range of mass flow rates and extraction pressures.

Cogeneration targets in process integration have been approached in various ways that go from the use of the graphical determination to the more elaborate models. The potential for cogeneration in a process can be approximated by representing the Grand Composite Curve (Linnhoff et al., 1982) in terms of the Carnot factor; this gives rise to the Exergy Grand Composite Curve (Dhole and Linnhoff, 1993). For large industrial complexes, a tool known as Total Site Composite Curve has been developed and this can be represented in terms of the Carnot factor as well (Klemes et al., 1997). The exergy model is based on the fact that for any heat transfer process, the area enclosed between a hot source and a cold sink is proportional to the exergy loss; however, if an ideal thermal engine (Carnot cycle) is placed in between, power will be generated. When the various steam levels that supply heat to a process are plotted on the Exergy Grand Composite Curve, the total area enclosed between these levels is related to the potential for shaft work production (Figs. 1 and 2). This model provides an easy and straightforward method to target for shaft work potential; however, a real steam turbine (Rankine cycle) exhibits a different performance and predictions based on the Carnot cycle are not accurate.

In an attempt to eliminate the limitations of the ideal models based on exergy calculations, Raissi (1994) proposed a method to predict the shaft work production from single pass out turbines. The most important feature of this work is that the power output is assumed to be proportional to the mass flow rate and the difference of saturation temperatures between inlet and outlet conditions. This model does not incorporate the consideration of the variation of the thermal efficiency with turbine size or with the operating load. The work by Mavromatis and Kokossis (1998) overcomes these limitations; it is based on the assumption that the performance of a steam turbine is described by a linear relationship known as Willans line; with this approximation the relations between turbine size and

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efficiency and even the variation of turbine efficiency with load are readily incorporated. Later, Varbanov et al. (2004) proposed a slight modification to this model that consists in the determination of the power output as a function of coefficients which in turn are calculated from linear equations that depend on the saturation temperature difference between inlet and outlet conditions.

This work presents a modified thermodynamic model for the prediction of the thermal performance of back pressure steam turbines with single and multiple extractions. The model makes use of some of the advantages of the turbine hardware model (Mavromatis and Kokossis, 1998). The nominal power production of commercially available steam turbines is used to validate the reliability of this model.

2. Modified thermodynamic model

From an energy balance, the power output (W) of a single extraction back pressure steam turbine can be determined from:

$$W = m(h_1 - h_2) \tag{1}$$

where m is the steam mass flow rate; h_1 and h_2 are the specific enthalpies at inlet and outlet conditions respectively. The enthalpy change $(h_1 - h_2)$ can be calculated using the expression for the isentropic efficiency:

$$\eta_{iso} = \frac{h_1 - h_2}{h_1 - h_{2iso}} \tag{2}$$

where, h_{2iso} is the outlet specific enthalpy of the steam considering isentropic conditions. Combining Eqs. (1) and (2) we obtain:

$$\frac{W}{\eta_{iso}} = m\Delta h_{iso} \tag{3}$$

where, Δh_{iso} is the isentropic enthalpy change. Peterson and Mann (1985) presented a set of curves that show the behaviour of the isentropic efficiency as a function of the power output for pass out turbines. The curves correspond to different inlet pressures (in the range of 1.4–10.2 MPa) for a constant inlet temperature of 540 °C and an outlet pressure of 0.1 MPa. The curves cover a range of power production of up to 100 MW. When these curves are transformed into linear relationships, they all can be correlated by a single linear equation:

$$\frac{W}{\eta_{iso}} = \alpha + \beta W \tag{4}$$

Combining Eqs. (3) and (4) we have:

$$W = \frac{1}{\beta}[m\Delta h_{iso} - \alpha] \tag{5}$$

where α and β are regression parameters that can be determined as a function of the pressure at the inlet condition. Mavromatis and Kokossis (1998), obtained correlations to determine the values of the parameters α and β. They presented two sets of equations depending on the range of power production and the correlating parameters were obtained as a function of the inlet saturation temperature. Varbanov et al. (2004) in a similar way presented two sets of equations to determine α and β with the difference that these were obtained as a function of ΔT_{sat}. However, from the information of Peterson and Mann (1985) we can derive a single correlation that covers the range of turbine sizes and where the correlating parameters are only a function of the inlet pressure since the outlet pressure is uniform for all cases.

$$\alpha = \psi + \varphi P^{in}$$

$$\beta = \gamma + \delta P^{in} \tag{6}$$

The values of ψ, φ, γ and δ are shown in Table 1. The term Δh_{iso} from Eq. (5) can be determined using the equation proposed

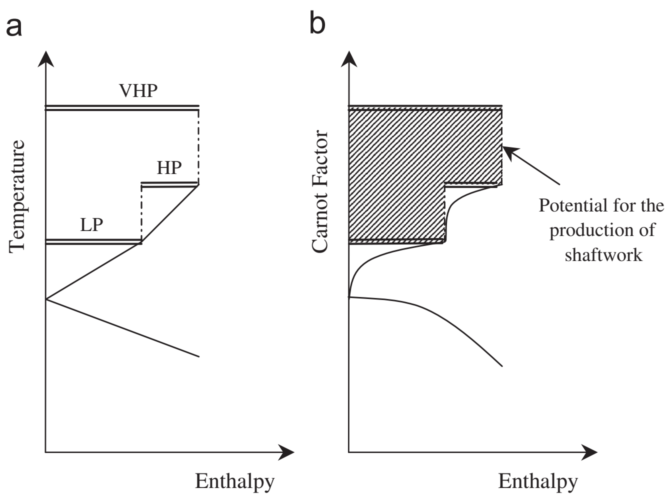

Fig. 1. (a) Grand Composite Curve and (b) Shaft work targets using the Exergy Grand Composite Curve (Linnhoff et al., 1982).

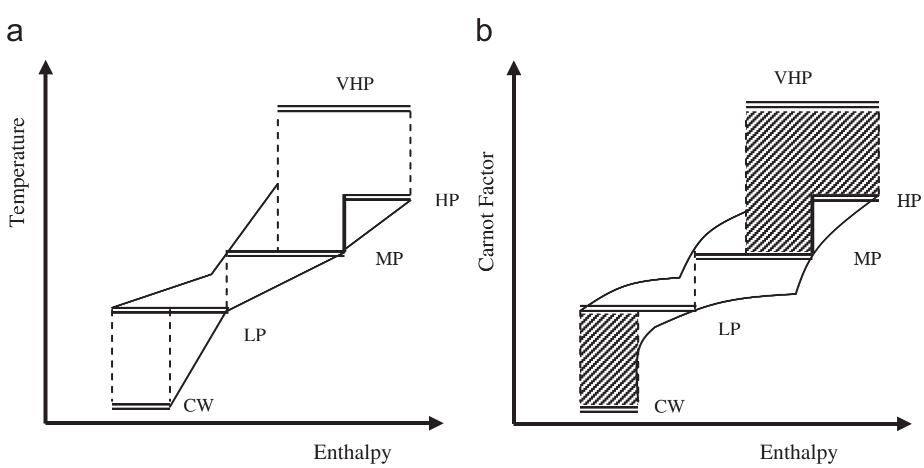

Fig. 2. (a) Total Site Composite Curve and (b) Exergy total Site Composite Curve (Dhole and Linnhoff, 1993).

by Singh (1997):

$$\Delta h_{iso} = \frac{\Delta T_{sat}}{1854 - 1931 q^{in}} \quad (7)$$

where ΔT_{sat} is the difference between the saturation temperature at inlet and outlet conditions. The term q^{in} is the specific enthalpy gained by the high pressure saturated liquid to reach the superheated condition before is fed to the turbine. Its value can be readily calculated once the inlet temperature and pressure are fixed.

Another parameter that needs to be determined is the heat load that the turbine delivers to the process (q^{out}). This term can be readily obtained by applying a heat balance around the Rankine cycle represented on a temperature-enthalpy diagram. Such a balance gives:

$$q^{out} = q^{in} + (h_{liquid,2} - h_{liquid,1}) - w \quad (8)$$

where $h_{liquid,1}$ and $h_{liquid,2}$ are the specific enthalpies of the saturated liquid at the conditions of low and high pressure respectively and w is the specific power output. The enthalpy difference $(h_{liquid,2} - h_{liquid,1})$ can be calculated from:

$$h_{liquid,2} - h_{liquid,1} = C_P \Delta T_{sat} \quad (9)$$

where the heat capacity of the liquid can be obtained from (Singh, 1997):

$$C_p = \mu_1 + \mu_2 T_{sat} \quad (10)$$

Eq. (9) is valid over a wide range of operating pressures going from 200 to 12000 kPa with maximum errors of $\pm$10%. In Eq. (10), T_{sat} is the temperature of saturation that corresponds to the turbine inlet pressure, μ_1=3.38 kJ/kg °C and μ_2=0.006123 kJ/kg °C^2. For the calculation of the power output W, an iterative procedure has to be implemented. This is diagrammatically shown in Fig. 3.

Table 1
Regression coefficients for parameters α and β.

Single extraction back pressure turbine	
ψ (MW)	0.1854
φ (MW/MPa)	0.0433
γ	1.2057
δ (MPa^{-1})	0.0075

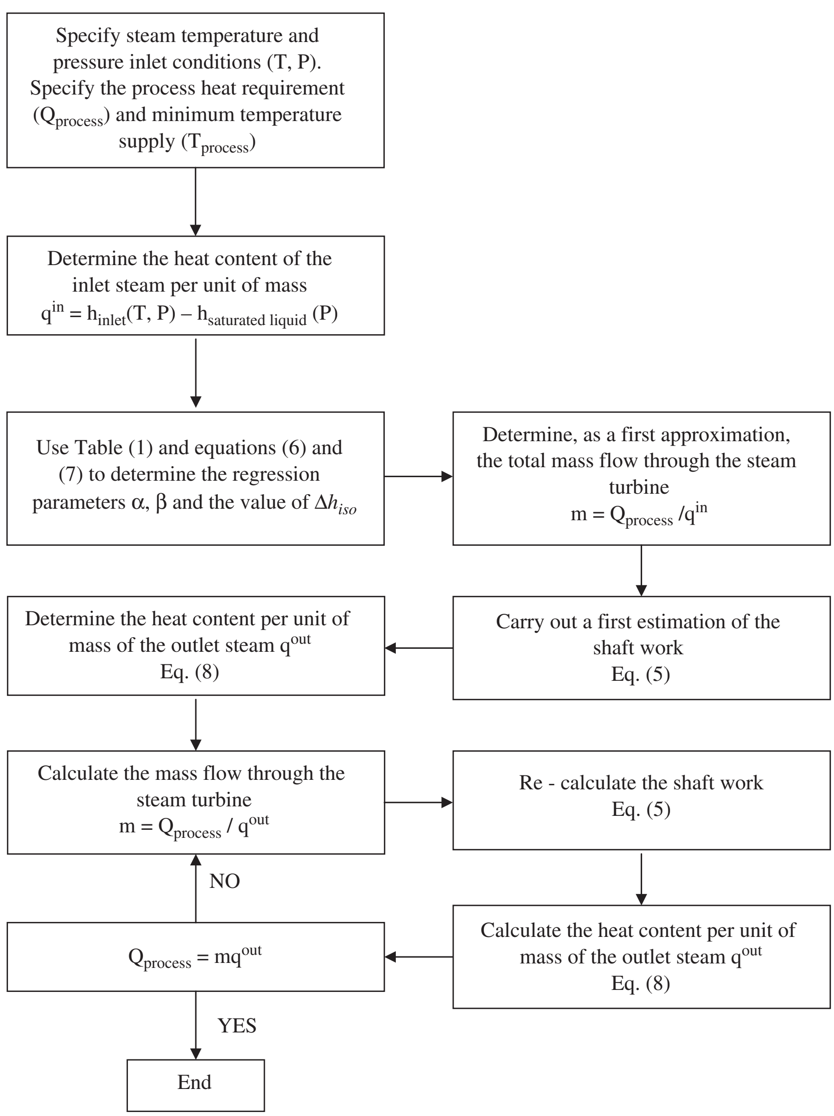

Fig. 3. Algorithm for the calculation of the power output in single extraction turbines.

Table 2
Performance parameters of three commercial steam turbines.

	Turbine 1[a] Model TM10000A	Turbine 2[b]	Turbine 3[c]
Inlet temperature (°C)	520	538	344
Inlet pressure (MPa)	8.5	11.3	4.8
Back pressure (MPa)	1.6	1.27	1.0
Steam flow rate (kg/s)	30	54.7	12.2
Nominal power output (MW)	11	25	3

[a] www.tgmturbinas.com.br.
[b] www.gepower.com.
[c] www.epa.gov.

Table 3
Comparison of theoretical models with actual commercial turbine performance.

Model	Turbine 1	Turbine 2	Turbine 3
Mavromatis and Kokossis (1998)			
Process heat load (MW)	67	124	24
Mass flow rate (kg/s)	29.74	55.51	11.51
Power (MW)	10.70	25.15	2.72
Error (%)	−2.80	+0.59	−10.29
Varbanov et al. (2004)			
Process heat load (MW)	67	124	24
Mass flow rate (kg/s)	30.15	56.13	11.71
Power (MW)	11.77	26.81	3.17
Error (%)	+6.54	+6.75	+5.36
New model			
Process heat load (MW)	67	124	24
Mass flow rate (kg/s)	29.84	55.47	11.63
Power (MW)	10.97	25.04	3.00
Error (%)	−0.27	+0.16	+0.0

3. Model validation

The power output that is obtained from the application of the theoretical model using operating data from commercial steam turbines is compared to the actual nominal power output. Two other models reported in the literature are also solved to compare their accuracy. These models are the ones presented by Mavromatis and Kokossis (1998) and Varbanov et al. (2004). Table 2 shows the performance parameters of three commercial turbines. The details of the results obtained using the three models are shown in Table 3.

From Table 3, it can be seen that all models give acceptable results, however, in most cases, the deviation from the power output of the real turbines is lower in the case of the new model.

4. Model for multiple extraction turbines

In cogeneration schemes with multiple extractions, the process itself dictates the operating variables such as: the number of extractions, the pressure of the extractions, and the mass flow rate of each extraction. This is, the process determines the number of steam levels and the minimum temperature at which they must be available and this is also in direct connection with the maximum power output that can be generated. The number of extractions can be as big as eight (Cotton, 1993), however it is common to use three extraction levels. These are referred to as: High pressure (HP), medium pressure (MP) and low pressure (LP) levels (Surinder et al., 1978). The thermodynamic model for multiple extractions presented here is based on the turbine model for single extraction developed in Section 2 and the model proposed by Lee (1954) and Chou and Shih (1987) where a multiple extraction turbine is modelled using a set of single turbines in series. A schematic is shown in Fig. 4. The algorithm used to determine the power production is shown in Fig. 5.

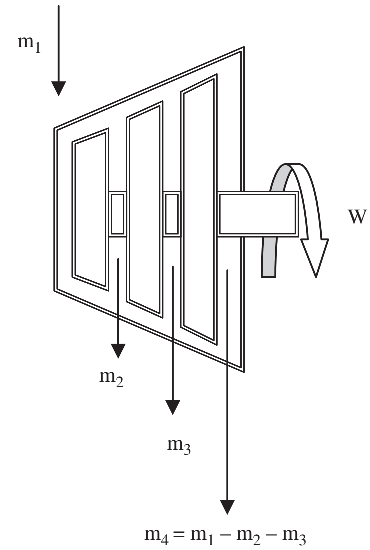

Fig. 4. Steam mass flow rates through the various sections of a multiple extraction turbine.

5. Validation of the multiple extraction turbine model

Performance data of multiple extraction steam turbines are taken from the open literature and shown in Table 4. The model by Mavromatis and Kokossis (1998) and Varbanov et al. (2004) are again used to compare with the new model. From the results shown in Table 5, it can be seen that the prediction using the new model exhibits the lowest deviations from the nominal power output.

6. Performance prediction under changed operating conditions

The shaft work produced by a turbine with N extractions is given by (Shapiro and Moran, 2004):

$$W=(h_1-h_{2r})\sum_{k=1}^{N}m_k+\sum_{j=2}^{N}\left\{\left[\sum_{k=j}^{N}m_k\right](h_{(k)r}-h_{(k+1)r})\right\} \tag{11}$$

where h_1 is the specific enthalpy of the steam fed to the turbine; h_{2r} is the specific enthalpy of the first extraction; $h_{(k)r}$ and $h_{(k+1)r}$ are the specific enthalpies at the inlet and outlet of extraction "k" and m_k is the steam mass flow rate in each of the expansion stages. The values of m_k are determined as a function of the heat load required by the process at each level.

$$m_k=\frac{Q_{process(k)}}{h_{(k)r}-h_{(k)f-process}} \tag{12}$$

where, $Q_{process(k)}$ is the heat load required by the process at extraction "k", and $h_{(k)f-process}$ is the specific enthalpy of the saturated liquid at the pressure conditions of extraction "k".

From Eq. (11) we see that we need the values of all the specific enthalpies and mass flow rates in each stage for the shaft work to be calculated; however, the specific enthalpies are related to the isentropic efficiency as given by Eq. (13):

$$h_{(k+1)r}=h_{(k)r}-\eta_{iso}(h_{(k)r}-h_{(k+1)iso}) \tag{13}$$

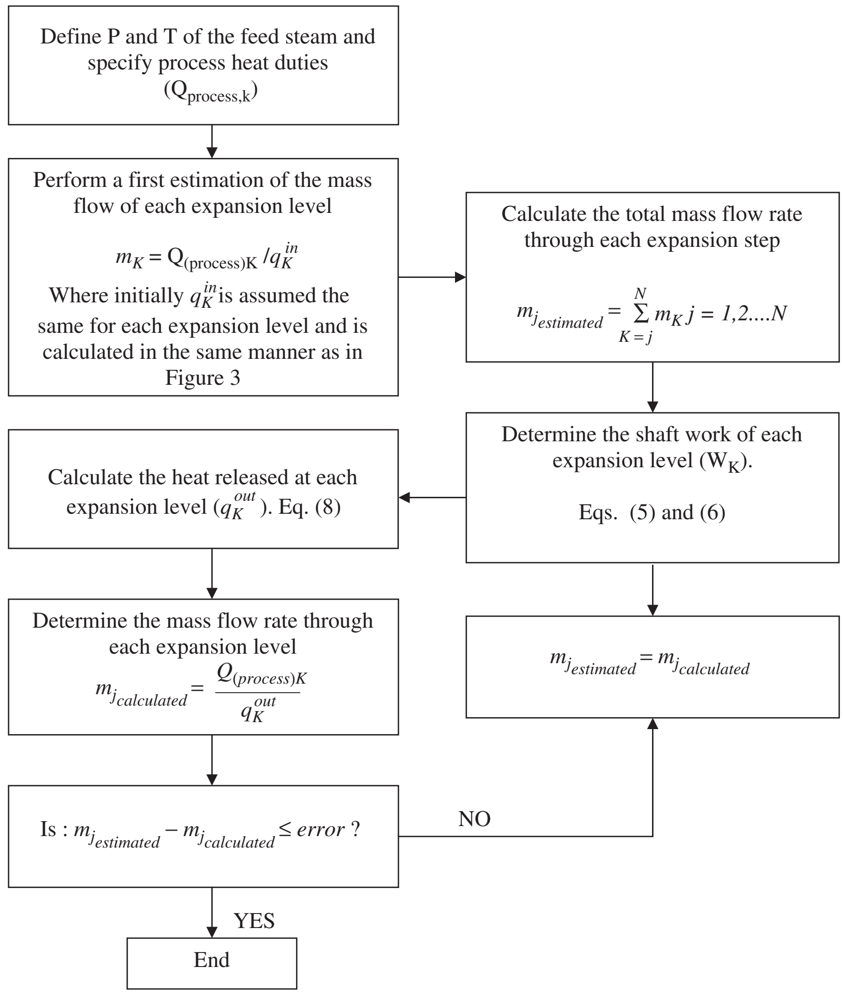

Fig. 5. Flow diagram for the determination of the power output of multiple extraction turbines.

where $h_{(k+1)iso}$ is the isentropic enthalpy at the extraction "k" and η_{iso} is the isentropic efficiency of that stage. The values of $h_{(k+1)iso}$ are a function of the extraction pressure and of the entropy at the temperature and pressure of the steam at the inlet of that stage as expressed by Eq. (14):

$$h_{(k+1)iso} = f(P_{extraction(k)}, s_{k-1}) \tag{14}$$

Eq. (13) contains two unknowns, η_{iso} and $h_{(k)r}$. The values of $h_{(k)r}$ can be obtained once the extraction pressure is fixed. This value can be fixed by knowing the typical range of operation of the various steam levels. According to Peterson and Mann (1985) these are: $5.5\,\text{MPa} \leq \text{HP} \leq 10\,\text{MPa}$, $0.8\,\text{MPa} \leq \text{MP} \leq 5.49\,\text{MPa}$ and $0.15\,\text{MPa} \leq \text{LP} \leq 0.79\,\text{MPa}$. Therefore, only the isentropic efficiency (η_{iso}) remains unknown in Eq. (13). The method presented in the sections above can be used to calculate its value.

Considering that the inlet operating conditions of turbines 4 and 5 of Table 4 remain unchanged (P=2.76 MPa and T=400 °C; P=6 MPa and T=440 °C) and the pressure of the extractions are allowed to vary, the isentropic efficiencies of the three expansion stages can be computed as described in Sections 2 and 4. The results from various scenarios chosen randomly are shown in Tables 6 and 7.

From the results of the Tables 6 and 7 it can be see that the turbine performance for the various scenarios could approximately be represented using a single weighted average efficiency (summation of power output times isentropic efficiency per stage, divided into the total turbine power) that lies between 0.722 and 0.76 for the first steam turbine; whilst 0.763 and 0.78 for the second one.

Table 4
Performance parameters of two commercial turbines with multiple extractions.

	Multiple extraction Turbine 4[a]	Multiple extraction Turbine 5[b]
Inlet temperature (°C)	400	440
Inlet pressure (MPa)	2.76	6.0
Pressure of first extraction (MPa)	1.07	1.23
Pressure of second extraction (MPa)	0.35	0.69
Back pressure (MPa)	0.12	0.27
Nominal power output (MW)	8	35

[a] www.gepower.com.
[b] www.toshiba.co.jp

In order to appreciate the error we are likely to incur in when a single uniform efficiency through the various stages is used, the shaft work of the steam turbines of Table 4 can be calculated using the model presented in this section (Eqs. (11)–(14)) and weighted average efficiencies. Tables 8 and 9 show the results of this exercise that covers a range of uniform efficiencies for the various scenarios of Tables 6 and 7. If a single isentropic efficiency of 0.74 is to be used for the steam turbine 4, the errors computed for its operation under different scenarios are: For the base case (+0.37%); for scenario 1 (+4%); for scenario 2 (−0.67%) and for scenario 3 (−2.54%). In the same way, if a single isentropic

Table 5
Power output of turbines with multiple extraction using different models.

Model	Parameter	Multiple extraction Turbine 4	Multiple extraction Turbine 5
Mavromatis and Kokossis (1998)	$Q_{\text{First extraction}}$ (MW)	31	90
	$Q_{\text{Second extraction}}$ (MW)	20	72
	$Q_{\text{Third extraction}}$ (MW)	9	30
	$m_{\text{First extraction}}$ (kg/s)	13.63	41.36
	$m_{\text{Second extraction}}$ (kg/s)	8.68	32.75
	$m_{\text{Third extraction}}$ (kg/s)	3.87	13.46
	$\eta_{\text{isentropic step I}}$	0.75	0.78
	$\eta_{\text{isentropic step II}}$	0.68	0.69
	$\eta_{\text{isentropic step III}}$	0.61	0.65
	$W_{\text{step I}}$ (MW)	5.09	28.59
	$W_{\text{step II}}$ (MW)	2.17	4.15
	$W_{\text{step III}}$ (MW)	0.463	1.61
	Power output (MW)	7.72	34.35
	Error (%)	−3.58	−1.89
Varbanov et al. (2004)	$Q_{\text{First extraction}}$ (MW)	31	90
	$Q_{\text{Second extraction}}$ (MW)	20	72
	$Q_{\text{Third extraction}}$ (MW)	9	30
	$m_{\text{First extraction}}$ (kg/s)	13.73	41.69
	$m_{\text{Second extraction}}$ (kg/s)	8.86	33.29
	$m_{\text{Third extraction}}$ (kg/s)	4.01	13.9
	$\eta_{\text{isentropic step I}}$	0.82	0.81
	$\eta_{\text{isentropic step II}}$	0.79	0.75
	$\eta_{\text{isentropic step III}}$	0.65	0.7
	$W_{\text{step I}}$ (MW)	5.65	30.53
	$W_{\text{step II}}$ (MW)	2.55	5.07
	$W_{\text{step III}}$ (MW)	0.64	2.19
	Power output (MW)	8.84	37.79
	Error (%)	+9.5	+7.38
New model	$Q_{\text{First extraction}}$ (MW)	31	90
	$Q_{\text{Second extraction}}$ (MW)	20	72
	$Q_{\text{Third extraction}}$ (MW)	9	30
	$m_{\text{First extraction}}$ (kg/s)	13.66	41.41
	$m_{\text{Second extraction}}$ (kg/s)	8.76	32.89
	$m_{\text{Third extraction}}$ (kg/s)	3.85	13.5
	$\eta_{\text{isentropic step I}}$	0.78	0.79
	$\eta_{\text{isentropic step II}}$	0.74	0.74
	$\eta_{\text{isentropic step III}}$	0.49	0.66
	$W_{\text{step I}}$ (MW)	5.27	28.91
	$W_{\text{step II}}$ (MW)	2.31	4.43
	$W_{\text{step III}}$ (MW)	0.36	1.61
	Power output (MW)	7.94	34.95
	Error (%)	−0.75	−0.14

Table 6
Isentropic efficiencies per stage for various operating scenarios (actual steam turbine 4).

	Base case	Scenario 1	Scenario 2	Scenario 3
$P_{1,\text{ First extraction}}$ (MPa)	1.07	1.5	0.8	0.5
$\eta_{\text{isentropic Step-I}}$	0.77	0.76	0.78	0.79
Mass flow rate$_{\text{, First extraction}}$ (kg/s)	13.66	13.68	13.64	13.61
$W_{\text{Step-I}}$ (MW)	5.27	3.44	6.75	8.95
$P_{2,\text{ Second extraction}}$ (MPa)	0.35	0.8	0.4	0.2
$\eta_{\text{isentropic Step-II}}$	0.73	0.69	0.68	0.7
Mass flow rate$_{\text{, Second extraction}}$ (kg/s)	8.76	8.75	8.74	8.75
$W_{\text{Step-II}}$ (MW)	2.31	1.37	1.31	1.6
$P_{3,\text{ Third extraction}}$ (MPa)	0.12	0.2	0.05	0.05
$\eta_{\text{isentropic Step-III}}$	0.48	0.59	0.63	0.53
Mass flow rate$_{\text{, Third extraction}}$ (kg/s)	3.85	3.85	3.87	3.86
$W_{\text{Step-III}}$ (MW)	0.36	0.65	0.88	0.45
Power output (MW)	7.94	5.46	8.94	11
Total mass flow rate	26.27	26.28	26.25	26.22
Isentropic efficiency (weighted average)	0.745	0.722	0.75	0.76

efficiency of 0.78 is to be used for the steam turbine 5, the errors computed for its operation under different scenarios are: For the base case (+0.14%); for scenario 1 (+2.72%); for scenario 2 (+0.237%) and for scenario 3 (+0.39%).

The application of a steam turbine model in optimization studies requires the determination of its performance under changed operating conditions. To this end an approximation can be implemented: Given the inlet steam operating conditions to the

Table 7
Isentropic efficiencies per stage for various operating scenarios (actual steam turbine 5).

	Base case	Scenario 1	Scenario 2	Scenario 3
$P_{1,\ \text{First extraction}}$ (MPa)	1.23	5.5	0.6	0.8
$\eta_{\text{isentropic Step-I}}$	0.79	0.65	0.79	0.79
Mass flow rate$_{\text{, First extraction}}$ (kg/s)	41.41	43.38	40.72	40.99
$W_{\text{Step-I}}$ (MW)	28.91	1.58	38.6	34.91
$P_{2,\ \text{Second extraction}}$ (MPa)	0.69	0.8	0.4	0.2
$\eta_{\text{isentropic Step-II}}$	0.74	0.78	0.7	0.77
Mass flow rate$_{\text{, Second extraction}}$ (kg/s)	32.89	32.77	32.46	32.55
$W_{\text{Step-II}}$ (MW)	4.43	17.5	2.64	9.47
$P_{3,\ \text{Third extraction}}$ (MPa)	0.27	0.05	0.2	0.05
$\eta_{\text{isentropic Step-III}}$	0.66	0.74	0.6	0.67
Mass flow rate$_{\text{, Third extraction}}$ (kg/s)	13.5	13.4	13.37	13.44
$W_{\text{Step-III}}$ (MW)	1.61	4.73	1.0	1.9
Power output (MW)	34.95	23.81	42.24	46.28
Total mass flow rate (kg/s)	87.8	89.55	86.55	86.98
Isentropic efficiency (weighted average)	0.777	0.763	0.78	0.78

Table 8
Power production for different operating scenarios and different overall average efficiencies (actual steam turbine 4).

	Expected power output (MW) (Turbine 4)			
	Base case (7.94 MW)	Scenario 1 (5.46 MW)	Scenario 2 (8.94 MW)	Scenario 3 (11 MW)
Power for $\eta_{\text{isentropic}}$=0.72	7.74	5.52	8.62	10.41
Power for $\eta_{\text{isentropic}}$=0.73	7.85	5.6	8.75	10.56
Power for $\eta_{\text{isentropic}}$=0.74	7.97	5.68	8.88	10.72
Power for $\eta_{\text{isentropic}}$=0.75	8.08	5.76	9.0	10.88
Power for $\eta_{\text{isentropic}}$=0.76	8.2	5.84	9.14	11.04
Power for $\eta_{\text{isentropic}}$=0.77	8.31	5.93	9.27	11.2

Table 9
Power production for different operating scenarios and different overall average efficiencies (actual steam turbine 5).

	Expected power output (MW) (Turbine 5)			
	Base case (34.95 MW)	Scenario 1 (23.81 MW)	Scenario 2 (42.28 MW)	Scenario 3 (46.28 MW)
Power for $\eta_{\text{isentropic}}$=0.75	33.5	23.37	40.5	44.4
Power for $\eta_{\text{isentropic}}$=0.76	34	23.73	41.11	45.1
Power for $\eta_{\text{isentropic}}$=0.77	34.5	24.1	41.74	45.77
Power for $\eta_{\text{isentropic}}$=0.78	35	24.46	42.38	46.46
Power for $\eta_{\text{isentropic}}$=0.79	35.6	24.83	43	47.16

turbine and using an average isentropic efficiency, the enthalpies and temperatures of the various extractions can be calculated using Eqs. (13) and (14). The pressure of the extractions is allowed to vary between the operating range described by Peterson and Mann (1985). So, a set of data for enthalpy and temperature can be obtained for each extraction pressure. These values can be correlated in the form given by Eqs. (15) and (16):

$$T(P_{extraction}) = \lambda_1 P_{extraction}^2 + \lambda_2 P_{extraction} + \lambda_3 \quad (15)$$

$$h(P_{extraction}) = \lambda_4 P_{extraction}^2 + \lambda_5 P_{extraction} + \lambda_6 \quad (16)$$

The functions above can be applied to the following variables: The specific enthalpy at outlet conditions, the specific saturation enthalpy at the pressure of the extraction, the temperature of the extraction and the saturated temperature at the pressure of the extraction. The correlating parameters λ_1, λ_2, λ_3, λ_4, λ_5 and λ_6 of Eqs. (15) and (16) depend on the pressure and the temperature of the steam entering the turbine (see Appendix A), so they have to be determined for each particular turbine feed conditions. As an example, Tables 10 and 11 show the results for these parameters for the inlet operating conditions specified.

Table 10
Coefficients of quadratic equation of the form given by Eq. (15).

Function	λ_1 (°C/kPa^2)	λ_2 (°C/kPa)	λ_3 (°C)
TV_{MP}	−0.000007	0.0842	221.88
TV_{MPsat}	−0.000003	0.0385	145.71
TV_{LP}	−0.00003	0.4467	85.53
TV_{LPsat}	−0.0001	0.1761	89.19
Function	λ_4 (kJ/kg kPa^2)	λ_5 (kJ/kg kPa)	λ_6 (kJ/kg)
h_{MP}	−0.00001	0.1566	2870.29
h_{MPsat}	−0.00001	0.1704	583.31
h_{LP}	−0.0005	0.8334	2639.67
h_{LPsat}	−0.0004	0.749	369.04

Steam turbine inlet conditions: T_{inlet}=400 °C, P_{inlet}=2.76 MPa, h_{inlet}=3235 kJ/kg and $\eta_{\text{weighted average isentropic}}$=0.74.

Table 11
Coefficients of quadratic equation of the form given by Eq. (15).

Function	λ_1 (°C/kPa^2)	λ_2 (°C/kPa)	λ_3 (°C)
TV_{MP}	−0.000006	0.0798	168.47
TV_{MPsat}	−0.000003	0.0385	145.71
TV_{LP}	−0.00008	0.2537	72.23
TV_{LPsat}	−0.0001	0.1761	89.19
Function	λ_4 (kJ/kg kPa^2)	λ_5 (kJ/kg kPa)	λ_6 (kJ/kg)
h_{MP}	−0.00001	0.1424	2785.08
h_{MPsat}	−0.00001	0.1704	583.31
h_{LP}	−0.0004	0.7399	2546.12
h_{LPsat}	−0.0004	0.749	369.04

Steam turbine inlet conditions: T_{inlet}=440 °C, P_{inlet}=6 MPa, h_{inlet}=3277 kJ/kg and $\eta_{\text{weighted average isentropic}}$=0.78.

In order to assess the accuracy of the application of the correlations represented by the Eqs. (15) and (16), the power production of the steam turbines is determined, for the four scenarios (Tables 8 and 9), using Eqs. (11) and (12) and the correlating parameters given in Tables 10 and 11. Tables 12 and 13 show the results of the calculation, and as can be seen, they indicate that a good approximation is achieved.

The search of the optimum extraction levels in a single process requires the turbine to be tested under changed extraction levels; for this to be achieved, a flexible model is required. A model like the one developed here allows for the calculation of the power output under changed operating conditions. This kind of model can be used in optimization studies.

7. Conclusions

The thermodynamic model presented in this paper aims at reconciling two important aspects related to the design of cogeneration systems using steam turbines and its thermal integration into background processes. These two aspects are: targeting the power production ahead of design and the selection of the actual turbine to be implemented. A targeting model must be accurate enough to predict the performance of a given turbine. This accuracy can be validated by comparing the results of the thermodynamic model with the ISO performance of commercial

Table 12
Power production of multiple extraction turbine 4 using the correlating parameters from Table 10.

	Base case	Results using parameters from Table 10 for Base case	Scenario 1	Results using parameters from Table 10 for Scenario 1	Scenario 2	Results using parameters from Table 10 for Scenario 2	Scenario 3	Results using parameters from Table 10 f or Scenario 3
$P_{1,\ \text{First extraction}}$ (MPa)	1.07	1.07	1.5	1.5	0.8	0.8	0.5	0.5
$\eta_{\text{isentropic, Step-I}}$	0.77	0.74	0.76	0.74	0.78	0.74	0.79	0.74
Mass flow rate$_{\text{, First extraction}}$ (kg/s)	13.66	13.64	13.68	13.68	13.64	13.62	13.61	13.60
$W_{\text{Step-I}}$ (MW)	5.27	5.48	3.44	4.01	6.75	6.46	8.95	7.59
$P_{2,\ \text{Second extraction}}$ (MPa)	0.35	0.35	0.8	0.8	0.4	0.4	0.2	0.2
$\eta_{\text{isentropic, Step-II}}$	0.73	0.74	0.69	0.74	0.68	0.74	0.7	0.74
Mass flow rate$_{\text{, Second extraction}}$ (kg/s)	8.76	8.74	8.75	8.79	8.74	8.74	8.75	8.75
$W_{\text{Step-II}}$ (MW)	2.31	1.98	1.37	1.22	1.31	1.22	1.6	2.03
$P_{3.\ \text{Third extraction}}$ (MPa)	0.12	0.12	0.2	0.2	0.05	0.05	0.05	0.05
$\eta_{\text{isentropic, Step-III}}$	0.48	0.74	0.59	0.74	0.63	0.74	0.53	0.74
Mass flow rate$_{\text{, Third extraction}}$ (kg/s)	3.85	3.95	3.85	3.94	3.87	3.95	3.86	3.95
$W_{\text{Step-III}}$ (MW)	0.36	0.54	0.65	0.78	0.88	0.84	0.45	0.42
Power output (MW)	7.94	8	5.46	6.01	8.94	8.52	11	10.04

Table 13
Power production of multiple extraction turbine 5 using the correlating parameters from Table 11.

	Base case	Results using parameters of Table 11 for Base case	Scenario 1	Results using parameters of Table 11 for Scenario 1	Scenario 2	Results using parameters of Table 11 for Scenario 2	Scenario 3	Results using parameters of Table 11 for Scenario 3
$P_{1,\ \text{First extraction}}$ (MPa)	1.23	1.23	5.5	5.5	0.6	0.6	0.8	0.8
$\eta_{\text{isentropic, Step-I}}$	0.79	0.78	0.65	0.78	0.79	0.78	0.79	0.78
Mass flow rate$_{\text{, First extraction}}$ (kg/s)	41.41	41.53	43.38	43.95	40.72	41.19	40.99	41.3
$W_{\text{Step-I}}$ (MW)	28.91	29.4	1.58	1.06	38.7	36.17	34.91	33.93
$P_{2,\ \text{Second extraction}}$ (MPa)	0.69	0.69	0.8	0.8	0.4	0.4	0.2	0.2
$\eta_{\text{isentropic, Step-II}}$	0.74	0.78	0.78	0.78	0.7	0.78	0.77	0.78
Mass flow rate$_{\text{, Second extraction}}$ (kg/s)	32.89	33.17	32.77	33.18	32.46	33.13	32.55	33.1
$W_{\text{Step-II}}$ (MW)	4.43	3.7	17.5	18.02	2.58	4.16	9.47	10.05
$P_{3.\ \text{Third extraction}}$ (MPa)	0.27	0.27	0.05	0.05	0.2	0.2	0.05	0.05
$\eta_{\text{isentropic, Step-III}}$	0.66	0.78	0.74	0.78	0.6	0.78	0.67	0.78
Mass flow rate$_{\text{, Third extraction}}$ (kg/s)	13.5	13.8	13.4	13.78	13.37	13.79	13.44	13.78
$W_{\text{Step-III}}$ (MW)	1.61	2.06	4.73	4.13	1.0	1.38	1.9	1.32
Power output (MW)	34.95	35.16	23.81	23.21	42.28	41.71	46.28	45.3

turbines; such are the features of the model presented in this work. When it comes to the thermal integration of steam turbines into background processes, an additional aspect that must be considered is the one related to the determination of the performance under changed operating conditions. For the case of multiple extraction turbines, it is desirable to determine how a steam turbine performs under the following scenarios: (a) variation of number of extractions, (b) variation of the extraction pressure and (c) variation of the steam mass flow rate of each extraction. The approach developed here is based on the determination of expressions that correlate steam temperature and enthalpy as a function of the extraction pressure and the determination of an overall mean isentropic efficiency; these elements are used to simulate the turbine performance under the various operating conditions. However, there is still a need to find the means of validating the model. An approach that can be suggested consists in the collection of performance data from multiple extraction turbines in operation against which the accuracy of the model predictions can be determined. A model thus validated can be used in heat and power optimization studies.

Notation

Cp	specific heat capacity, kJ/kg °C
h_1	specific enthalpy of steam at inlet conditions, kJ/kg
h_2	specific enthalpy of steam at first extraction, kJ/kg
h_{2iso}	isentropic specific enthalpy of steam at first extraction, kJ/kg
h_{2r}	actual specific enthalpy of steam at first extraction, kJ/kg
h_{HP}	specific enthalpy of steam at high pressure, kJ/kg
h_{HPsat}	specific enthalpy of saturated steam at high pressure, kJ/kg
h_{LP}	specific enthalpy of steam at low pressure, kJ/kg
h_{LPsat}	specific enthalpy of saturated steam at low pressure, kJ/kg
h_{MP}	specific enthalpy of steam at medium pressure, kJ/kg
h_{Mpsat}	specific enthalpy of saturated steam at medium pressure conditions, kJ/kg
$h_{(k)r}$, $h_{(k+1)r}$	specific enthalpies at inlet and outlet conditions of extraction "k", kJ/kg
$h_{(k+1)iso}$	specific isentropic enthalpy at extraction "k", kJ/kg

$h_{(k)f-process}$ specific enthalpy of saturated liquid at the extraction pressure "k", kJ/kg
$h_{liquid,2}$ specific enthalpy of saturated liquid at high pressure extraction, kJ/kg
$h_{liquid,1}$ specific enthalpy of saturated liquid at low pressure extraction, kJ/kg
HP high pressure steam, MPa
Δh_{iso} isentropic enthalpy change
LP low pressure steam, MPa
m mass flow rate, kg/s
m_j total mass flow rate through each expansion step, kg/s
m_K first estimation of the mass flow of each expansion step, kg/s
MP medium pressure steam, MPa
N number of expansion levels
P pressure, MPa
$Q_{process}$ process heat load, MW
q^{in} specific enthalpy gain of high pressure saturated liquid, kJ/kg
q^{out} heat load released to the process per unit of mass, kJ/kg
s specific entropy, kJ/kg
$T_{process}$ process minimum temperature supply, °C
T_{sat} saturation temperature of steam at feed conditions, °C
TV_{HP} temperature of the steam at high pressure condition, °C
TV_{HPsat} saturation temperature of the steam at high pressure condition, °C
TV_{LP} temperature of the steam at low pressure condition, °C
TV_{LPsat} saturation temperature of the steam at high pressure condition, °C
TV_{MP} temperature of the steam at medium pressure condition, °C
TV_{MPsat} saturation temperature of the steam at medium pressure condition, °C
ΔT_{sat} difference in saturation temperature between inlet and outlet conditions, °C
W shaft work, MW
W_K first approximation for shaft work at expansion step "k", MW
w specific shaft work, kJ/kg

Greek Symbols

α regression parameter
β regression parameter
γ coefficients of linear correlation
δ coefficients of linear correlation, MPa^{-1}
η efficiency
η_{iso} isentropic efficiency
λ_1 coefficient of quadratic correlation, °C/kPa^2
λ_2 coefficient of quadratic correlation, °C/kPa
λ_3 coefficient of quadratic correlation, °C
λ_4 coefficient of quadratic correlation, kJ/kg kPa^2
λ_5 coefficient of quadratic correlation, kJ/kg kPa
λ_6 coefficient of quadratic correlation, kJ/kg
μ_1 coefficient of linear correlation, kJ/kg °C
μ_2 coefficient of linear correlation, kJ/kg °C^2
φ coefficients of linear correlation, MW/MPa
ψ coefficients of linear correlation, MW

Subscripts

1 turbine steam feed
2 first extraction
3 second extraction
4 third extraction

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Appendix A

Given the inlet steam operating conditions of the actual steam turbine, Eqs. (13) and (14) and an average isentropic efficiency are used to determine a set of data for enthalpy and temperature for a range of extraction pressures. This is shown in Table A.1 for the case of turbine 5 of Table 4. The pressure of the extractions is allowed to vary between the operating range described by Peterson and Mann (1985). The values of Table A.1 can be correlated in the form of Eq. (15) as shown in Figs. A.1 and A.2.

Table A.1
Predicted thermodynamic properties of steam at the various extraction levels Inlet operating conditions as given in Table 4 for turbine 5.

P_{MP} (kPa)	T_{MP} (°C)	T_{MPsat} (°C)	h_{MP} (kJ/kg)	h_{MPsat} (kJ/kg)	P_{LP} (kPa)	T_{LP} (°C)	T_{LPsat} (°C)	h_{LP} (kJ/kg)	h_{LPsat} (kJ/kg)
5400	426	268.8	3252	1179	720	212.4	166.1	2871	702.2
5200	421.1	266.4	3244	1167	650	203.6	162	2855	684.3
4750	409.4	260.8	3223	1138	550	189.3	155.5	2829	656
4500	402.5	257.5	3211	1122	525	185.1	153.7	2821	648.3
4250	395.4	254	3198	1105	500	180.8	151.8	2813	640.3
4000	387.9	250.4	3184	1087	475	176.3	149.9	2805	632
3750	380	246.6	3170	1069	450	171.6	147.9	2797	623.3
3500	371.7	242.6	3156	1050	425	166.7	145.8	2788	614.3
3250	363	238.4	3140	1029	400	161.6	143.6	2778	604.8
3000	353.7	233.9	3123	1008	375	156.2	141.3	2768	594.9
2750	343.7	229.1	3106	985.8	350	150.5	138.9	2758	584.4
2500	333.1	224	3087	961.9	325	144.5	136.3	2747	573.3
2250	321.6	218.4	3066	936.3	300	138.2	133.5	2735	561.5
2000	309.1	212.4	3044	908.6	275	131.4	130.6	2723	548.9
1750	295.2	205.7	3019	878.4	250	127.4	127.4	2710	535.4
1500	279.8	198.3	2991	844.8	225	124	124	2695	520.7
1250	262.3	189.8	2960	806.8	200	120.2	120.2	2680	504.7
1000	241.7	179.9	2924	762.8	175	116	116	2663	487
800	222.2	170.4	2889	721.1	150	111.4	111.4	2645	467.1

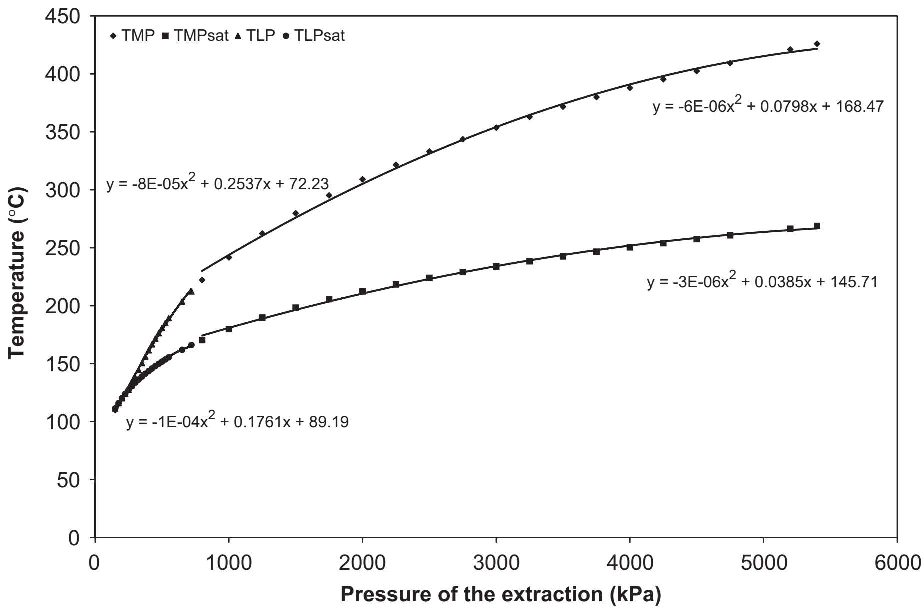

Fig. A.1. Temperature parameters for various extraction levels.

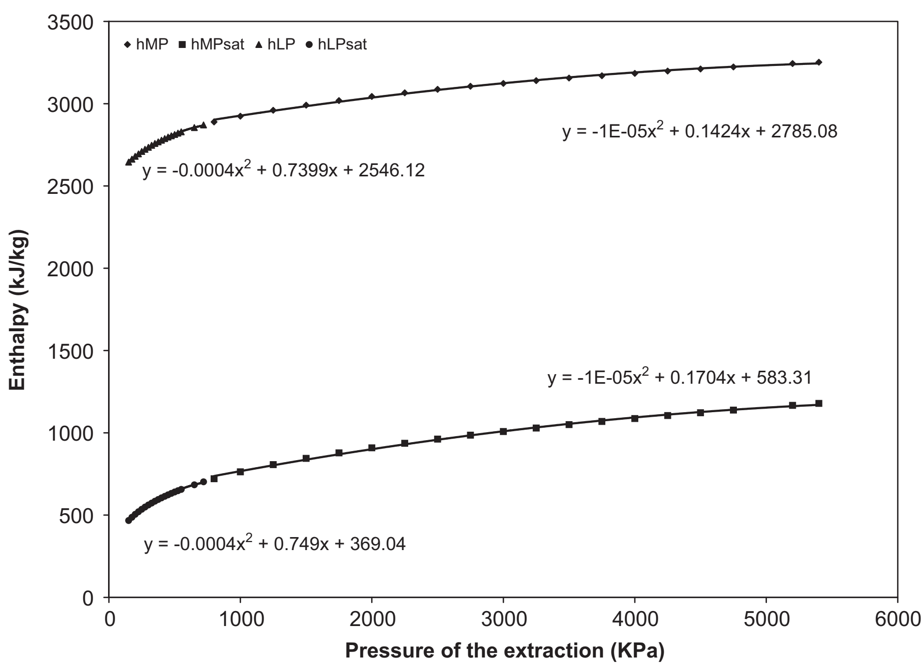

Fig. A.2. Enthalpy parameters for various extraction levels.

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