Active heat transfer enhancement in air cooled heat sinks using integrated centrifugal fans

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Abstract

The enhancement of convective heat transfer in an air-cooled heat sink using integrated, interdigitated impellers was investigated. The experimentally investigated heat sink is representative of a subcomponent of an unconventional heat exchanger with a loop heat pipe, multiple parallel flat-plate condensers, and integrated, interdigitated centrifugal fans, designed to meet the challenges of thermal management in compact electronic systems. The close integration of impeller blades with heat transfer surfaces results in a decreased thermal resistance per unit pumping power compared to conventional forced convection heat sinks.

The fan performance (i.e. fan curve and power consumption) and heat transfer of a single integrated fan heat sink were experimentally characterized for 12 impeller designs and modeled in terms of dimensionless correlations. Correlations were developed to give estimates of the dimensionless fan curve and the dimensionless power curve based on the fan geometry. Additionally, a two-parameter correlation was developed to estimate the dimensionless heat flux based on the fan’s operating point. The heat transfer in the integrated fans was observed to be a function of the operating point (i.e. the rotational speed of the impeller and the flow rate of air), with only a weak direct dependence on the fan geometry. The insensitivity of the heat transfer performance to the impeller geometry greatly simplifies the design process of integrated fan heat sinks because the fan design can be optimized independently of the heat transfer performance. Finally, the heat transfer enhancement (compared to pressure-driven flow at the same flow rate) appears to be due to turbulent flow structures induced by the impeller.

1. Introduction

1.1. Motivation

Modern electronics and computing power depend on effective thermal management to achieve high levels of performance. Considering the economic impact, it is not surprising that computer users and manufacturers are interested in state-of-the-art thermal management technologies. The International Electronics Manufacturing Initiative (iNEMI) roadmap [1] states that new processor technology will demand improved cooling and high density packaging technologies. In addition, the International Technology Roadmap for Semiconductors (ITRS) [2] predicts that the required thermal resistance for high performance ICs will decrease from its 2009 level of 0.27 K/W to 0.08 K/W in 2024, a more than threefold reduction. This forecast predicts that current air cooling technology will be incapable of meeting the increasingly stringent demands of the ICs: “The high junction-to-ambient thermal resistance resulting from an air-cooled heat sink provides inadequate heat removal capability at the necessary junction temperatures for ITRS projections at the end of this roadmap”.

Clearly, there is a broadly recognized need for more effective cooling — in particular, air cooling. The iNEMI roadmap also points out another important issue in thermal management solutions: heat sinks are subject to volume constraints. Minimum volume traditionally comes at the expense of thermal performance. These volume constraints are particularly evident in high end applications (e.g. telecommunications, radar, sensing and imaging), which can have a larger (~10x) thermal duty than consumer CPUs, but still be constrained to a similar volume envelope. Many such applications have resorted to “exotic liquid-cooled manifolds, spray-cooled enclosures, and vapor-compression refrigeration” due to the insufficient performance of the current state-of-the-art air cooling solutions [3]. With improvements in air cooling, simpler heat sinks could replace these bulky and complex solutions.
Cooling of electronics also has a surprisingly high energy cost, which translates to monetary cost and environmental impact. Meijer reported that data centers accounted for about 2% of worldwide energy consumption in 2009, and half of this was devoted to cooling [4]. Koomey estimates that the power use for data centers in 2010 accounted for 1.1–1.5% of worldwide electricity use, about half of which was used for cooling [5]. Any improvements in the efficiency of thermal management solutions would help to reduce some of this energy use.

Finally, compact and efficient air cooling solutions are critical for enabling proliferation of new technologies such as solid-state lighting. With increased adoption of these technologies and the continued pervasiveness of computing technology, any improvements in air cooling have the potential to effect appreciable energy savings with associated financial and energy security benefits.

1.2. Background literature

Conventional finned heat sinks have been studied by many investigators. Tuckerman and Pease [6] studied a microchannel heat sink etched on silicon and minimized thermal resistance by choosing the appropriate channel width, fin width and aspect ratio subject to constraints on the geometry and pressure drop. Knight et al. [7] developed an analytical method to minimize thermal resistance of a heat sink with pressure driven flow in a closed finned channel by varying the geometry. Teertstra et al. [8] developed an analytical model to calculate the Nusselt number for a plate fin heat sink as a function of geometry and flow properties. Bejan [9] illustrated a heat sink optimization using entropy generation minimization (EGM), a method whereby the entropy production is quantified and ascribed to the various loss producing mechanisms, and the free parameters are altered to minimize the overall entropy generation rate (for an overview of the method, see [10]). Culham and Muzychka [11] presented a method similar to that of Bejan, where the geometry of plate fin heat sinks is optimized by minimizing the entropy generation rate. Their method also allowed for the incorporation of real fan performance data in the model. Similarly, Khan et al. [12] applied EGM techniques to optimize a pin fin heat sink. Bar-Cohen and Iyengar [13,14] presented methods of optimizing a parallel plate heat sink. Their methods accounted for the energy used in the manufacture of the heat sink and the cooling energy used over the expected life of the computer in which it resides; their optima sometimes differed from the EGM optima, since the latter did not account for manufacturing energy. Furukawa and Yang [15] performed a numerical analysis of a plate fin heat sink in natural convection, and found their optimum design predictions to be very close to those of Bejan, Culham and Muzychka, and others.

Several recent studies have shown that finless designs can provide improved performance in small heat sinks. Egan et al. [16] looked at a miniature, low profile heat sink with and without fins and used particle image velocimetry to detail flow structures and heat transfer. Stafford et al. [17] studied forced convection cooling on low profile heat sinks with and without fins and showed that heat transfer rates of the finless designs were better than their finned counterparts.

A fundamentally different approach to air cooling was developed by Koplow [18]. The “Sandia Cooler”, or “air bearing heat exchanger” (ABHE), consists of a rotating disc atop a circular stator plate. The top of the disc has fins that extend upward and act as impeller blades to draw air in axially and discharge it radially. The air bearing between the disc and the stator has a low thermal resistance due to its thinness and large area. The ABHE exploits the slow boundary layer development that occurs in an accelerating reference frame (a phenomenon studied experimentally by Cobb in 1956 [19]). The performance of the Sandia Cooler is significantly better than traditional air cooled heat sinks on a per-unit-volume basis.

This work focuses on the air flow in a heat sink developed at MIT to address the needs of high performance electronics. This heat sink, referred to as “PHUMP”, consists of a loop heat pipe with multiple parallel condensers and integrated, interdigitated centrifugal fans [20–23]. The fan impellers drive air radially outwards across a multitude of parallel-plate condensers (Fig. 1). Heat transfer into the bottom of the device causes the working fluid, water, to evaporate and travel up the vertical pipes into the parallel condensers (the condensers are described in detail by Peters [24] and Hanks [21]). Heat is removed from the condensers by convective heat transfer to the air flow. Fresh, cool air is drawn in through the inlet at the top. This configuration exploits the tremendous heat transfer coefficient associated with evaporation to achieve a high heat flux at the evaporator base (the evaporator and system operation are described in detail by Kariya [20]). Although the heat transfer coefficient from the condensers to the air is much lower than in the evaporator, the large thermal power input from the evaporator is distributed among the parallel condensers. By parallelizing the condensers, their overall thermal resistance is greatly reduced, analogous to the lower equivalent resistance of electrical resistors in parallel. Finally, a practical advantage is that the device is self-contained, using the surrounding air as the heat exchange fluid and therefore requiring no external fluid connections.

The air flow design of the PHUMP also has advantages. First, a low profile motor can be used to drive the fans, which share a common shaft; this allows for a compact architecture that does not occupy much vertical space. Second, the fans actually enhance the convective heat transfer between the condenser surfaces and the airflow flowing through the device. Depending on the regime the fan is operating in, this heat transfer enhancement has been observed to be as high as 3× the equivalent flow rate of pressure-driven air flow. Finally, the integrated fan approach results in a device whose thermal resistance is very low compared to conventional heat sinks with a comparable volume. To illustrate this, the expected performance of an integrated fan heat sink that fits in a 102 mm cube (similar to the heat sink in Fig. 1) was compared to twenty commercially available air-cooled heat sinks (tested by Page [25]). The thermal resistance and volume of these commercial heat sinks are shown in Fig. 2. The commercial air-cooled heat sinks form a Pareto performance frontier, with the Sandia Cooler and PHUMP exceeding the frontier.

1.3. Objectives of present study

In the present study, we sought to accomplish the following:

- Experimentally measure the fan curves and power consumption of planar, fully unshrouded centrifugal fans that are well-suited to integration into a stack of planar heat pipe condensers.
- Experimentally measure the heat transfer in this integrated-fan system.
- Develop dimensionless correlations for these fan curves, power curves, and heat transfer coefficients.
- Develop an understanding of the trade-off between thermal performance and mechanical power consumption in these integrated-fan systems.

In Section 2, we discuss the prototype impeller/gap systems that were used and the experimental apparatus we used to map their performance. In Section 3, we describe the trends we observed in the system and try to reconcile them with several reasonable analyses. Dimensionless correlations are formulated based on the geometries characterized; the experimental data can be estimated with an accuracy of 20% or better relative root-mean-
squared error. These correlations allow thermal engineers to design integrated-fan systems without undertaking expensive and time-consuming CFD simulations or experiments.

2. Materials and methods

2.1. Integrated fan heat sink topology

The basic topology of a single-layer integrated fan heat sink is shown in Fig. 3, which shows a cutaway view with part of the top stator disc removed to show the impeller. This circular geometry was chosen (as opposed to a square geometry to mimic the device shown in Fig. 1) for characterization because of its simplicity; to account for additional flow restrictions such as support posts, the fundamental results of this circular geometry can be augmented with basic fluid mechanics and heat transfer analysis to obtain performance predictions.

In this basic integrated fan topology, air flow enters the axial inlet (of radius \( r_1 \) and diameter \( d_1 \)) and is pumped radially outward by the fan. As the air moves radially outward it passes over the hot surfaces until it is discharged at the outer periphery of the stator discs (of radius \( r_2 \) and diameter \( d_2 \)). The stator discs are separated by a gap of breadth \( b_2 \). The impeller blade has a breadth of \( b_n \), and is concentric with the stator discs, centered axially between them. The impeller has an exit angle of \( \beta_2 \), measured from the tangent (i.e. a radial blade has \( \beta_2 = 90^\circ \)).

The impeller rotates with an angular velocity \( \omega \), and develops a mass flow \( \dot{m} \) (volume flow \( V \)) and a total-to-static pressure rise of \( \Delta p_{t-s} = p_{a2} - p_{b0} \), where \( p_{a2} \) is the static pressure at the outlet and \( p_{b0} \) is the total pressure (i.e. stagnation pressure) at the inlet, upstream of where it enters the parallel plate gap. During operation the rotating fan requires a torque \( M \), and uses a mechanical power consumption of \( W = M \omega \).

The heat transfer into the system, \( \dot{Q} \), flows into the air from the uniform-wall-temperature \( T_w \) stator discs and causes the air temperature to increase above its inlet temperature \( T_{in} \). The air entering the system has a density, dynamic viscosity, and isobaric specific heat of \( \rho, \mu, \) and \( c_p \), respectively.

An experimental apparatus to characterize integrated fans is discussed below in Section 2.2. In this apparatus, the mass flow...
rate of air, the total-to-static pressure rise, the torque (mechanical power consumption), and heat transfer into the system were measured at various rotational speeds and for various levels of upstream flow restriction (i.e., for different flow rates). These measurements were used to determine fan curves, power curves, and heat transfer coefficient maps.

In the experimental apparatus, the air only entered the top axial inlet; the bottom inlet was present, but air flow was blocked by a cap. The pressure drop across the impeller in the core region was calculated to be small, and some care was taken to ensure that the impeller did not block too much of the axial inlet cross sectional area. If this area is impeded by less than about 50%, the effect of this blockage is expected to be quite small, even in multilayer devices with several of these integrated fans in an axial stack (for a thorough discussion of the scaling in multilayer devices, see Chapter 5 of Staats [26]).

2.2. Experimental apparatus

An apparatus was constructed to characterize the fundamental, generalized form of the fully unshrouded integrated fan, which consists of an impeller and two annular stator plates as shown in Fig. 3. This apparatus, shown in Fig. 4, directly measures the total-to-static pressure difference, because the pressure tap upstream of the fan is located in an area where the cross sectional area is large enough to make the flow's dynamic pressure negligible. The fan being characterized is the only force moving air through the apparatus, and thus flows at or beyond the free delivery flow rate cannot be characterized. To characterize the fan curve under restricted flow conditions, a throttle valve upstream of the fan restricted the flow in the system, effectively providing a higher system resistance for the fan to pump against. In this way, the fan's operation closely resembled the actual operating conditions that one would expect in a real flow system.

As shown in Fig. 4, air flow entered the system at the top, passed through an air filter (K&N R-1050) and then through a hot-wire anemometer type mass flow sensor (Eldridge Products, Inc. Master-Touch 8689MPNH-SSS-133, 1%+0.5%FS uncertainty). The air flow then passed through a modified automotive butterfly throttle body, the position of which was held by a servo (Turnigy MG90S) controlled by the DAQ computer. After the throttle, the air entered a cylindrical plenum whose entry contained a baffle plate to minimize jetting of the air flow from the throttle valve; a thermocouple situated on the baffle plate measured the temperature of the inlet air. A screen just downstream of the baffle plate served to smooth out small variations in the flow. Finally, the air in the plenum entered the parallel plate gap in which the fan impeller rotated, and was discharged to the ambient pressure surroundings.

A pressure tap in the plenum wall was located downstream of the screen. Two differential pressure sensors (Honeywell DC002NDC4, ±498 Pa, nominal 1.5% FS uncertainty; Honeywell ASDXLO5D44D, ±1245 Pa, nominal 2.5% FS uncertainty) were connected to the plenum pressure port and a static pressure tap at the outlet of the fan channel.

The motor that drove the fan was also situated within the plenum. A three-spoked, raised structure was mounted to the inside face of the plenum's bottom aluminum plate. The motor, bearing assembly, and shaft onto which the fans mounted were supported by this structure. By mounting the motor inside the plenum the stator plates and the impeller could be changed without disturbing the motor or bearings. The motor mount served to raise the motor far enough away from the fan's inlet that minimal air blockage occurred. The three motor mount standoffs were also placed far enough from the inlet that their effect on the inlet flow was negligible.

The impeller's rotation was driven by a DC motor (Maxon 339152). An integrated 500-count encoder (Maxon 225778) provided the speed measurement, and a controller (Maxon 390003) held the speed constant irrespective of the load torque on the impeller. The motor controller also provided a measurement of the electric current (torque) used by the motor. Both the control and measurement signals were exchanged via a USB connection to the data acquisition computer. The motor current had a considerable amount of variance during normal operation, presumably due to small variations in drag torque from the motor and shaft bearings. To reduce the uncertainty in the measured torque and rotational speed, each measurement was taken to be the mean value of 4000 samples. The standard deviation in the calculated motor power using this method was about 0.034 W (~5% of a typical mean value).

Several pieces of hardware were used to communicate between the experiment and the computer. A multifunction DAQ board (Measurement Computing USB-2537) with 32 differential 16-bit analog inputs read the voltage signals from the various sensors in the experiment. Two 16-bit, 16-channel analog output boards (Measurement Computing USB-3106) allowed the computer to control various aspects of the experiment (e.g., the throttle position and the power signals for the heated plates). A thermocouple input board (NI 9213 module in a NI cDAQ-9172 chassis) was used to read signals from the thermocouples in the system (located in the stator plates, Section 2.2.1, and on the baffle to measure the inlet air temperature). A more thorough description of the experimental apparatus can be found in Chapter 2 of Staats [26].

2.2.1. Heated stator plate design

The heat transfer coefficient of a fan was determined by running it at various operating points while heating the stator plates. The plates were heated to a constant temperature and held at this temperature by a closed-loop control system (PID control with filtered error signal). The electrical power required to maintain the plate temperature was measured at each operating point.

The copper heater plate assembly can be seen in detail views A and B of Fig. 4. Each stator plate consisted of two 3.2 mm thick
annular plates of copper ($d_1 = 40 \text{ mm}$ and $d_2 = 100 \text{ mm}$). Four polyimide film heaters (Electro-Flex Heat KH-0.5X2-10-28-A) were bonded to each impeller-adjacent copper plate. The space surrounding the rectangular-shaped heaters was filled by an additional copper piece of the same thickness as the heaters. Next, the remaining annular plate was adhered to the top of the film heaters and copper filler pieces with silver-filled thermally conductive epoxy. Thermocouple wells were formed by drilling small (~1 mm) diameter holes in two locations about 90° apart on the side of the impeller-adjacent copper plates. The outer edge of each stator plate assembly was epoxied to a thin polyethersulnone frame, which provided structural support and thermal insulation (with minimal contact with the copper assembly to reduce the heat leak to the surroundings). The upper polyethersulnone frame structure had a bolt pattern to mount to the lower face of the plenum.

Additionally, three equispaced dowel pins were rigidly mounted to the upper heater plate assembly. These pins mated to a matching hole pattern in the lower heater plate assembly, allowing it to slide axially while remaining coaxial with the upper plate. The gap spacing between the inner-facing copper surfaces was set by (1) axially sliding the lower heater plate assembly, (2) inserting feeler gauges of the desired thickness between the copper surfaces of the upper and lower heater assemblies, and (3) tightening cross-drilled thumb screws on the lower heater plate assembly to rigidly fix its location to the dowel pins. This gap-setting procedure was performed with the plates at the desired temperature to avoid movement due to thermal expansion.

The stator plate assemblies were designed to have isothermal copper regions so that the heat loss to ambient would occur from the same temperature as that measured by the thermocouples. With the plates at uniform temperature, the heat loss to ambient is independent of the heat flux into the air flow stream. The copper stators were assumed to be isothermal because the Biot number was very low (Bi $< 10^{-3}$). A finite element model of the plates also confirmed that there were negligible temperature gradients in the lateral direction of the copper, and that the copper satisfactorily spread the heat input of the heaters. Second, the heat leak from the copper to the ambient was assumed to originate from the same temperature. By embedding the heaters in the copper plates this assumption is much more valid compared to installing the heaters on the top side of a single copper plate. In this hypothetical case, the heat loss to ambient would occur from a higher temperature than that of the plate, due to thermal resistance between the heater and the plate.

The heaters in the stator plates were driven by a custom built linear amplifier to minimize noise introduction into the system. The voltage applied to the heaters (all four heaters of each plate were wired in parallel) was measured at the DAQ using a resistive divider.

2.3. Fan performance characterization

A fan performance characterization subjected a fan to a sequence of rotational speed set points and a sequence of throttle valve set points to yield data at 9 rotational speeds, ranging from the shut-off condition (no net flow) to as-near-as-possible to the free-delivery condition (minimal system resistance). The sequence of rotational speeds was $[1, 3, 5, 7, 9, 8, 6, 4, 2] \text{ krpm}$. The sequence of throttle positions was $[0, 0.1, 0.18, 0.3, 0.4, 1, 0.35, 0.25, 0.225, 0.15, 0.125, 0.075]$, where 0 and 1 correspond to fully closed and fully open, respectively. These sequences of set-points followed an up-then-down pattern to make hysteresis evident in post processing. This test procedure, performed for each fan test, yielded the pressure sensor readings, the mass flow rate, the current used by the motor, and the rotational speed at each operating point. This data was subsequently processed (the appropriate calibration curves were applied to the raw data) into meaningful operating parameters. Additionally, to account for the losses in the bearings, the torque (power) required to rotate the shaft with no impeller installed was measured over the operating range of rotational speeds. A parabolic fit to this power vs. rotational speed data (forced through the origin, since $W = 0$ at $\omega = 0$) was used to subtract the parasitic bearing power loss from the measured power consumption of the fans at each test point.

A slightly different version of the experimental apparatus with simple, unheated stator plates was used for some of the fan performance characterizations. 6 unheated stators with different inner and outer diameters ($d_1 = [40, 50]$ and $d_2 = [75, 100, 120]$) were fabricated and tested (the diameter of the fan was always equal to the outer diameter of the stator plate; scaled versions of Fan 5 were used for the cases when $d_2 = [75, 120]$).

2.4. Heat transfer characterization

After installation of the fan impeller to be tested, the heat transfer characterization experiments were performed using the following procedure. First, the heaters were turned on while the impeller was held stationary and the throttle valve held closed. The system was allowed to reach the set point temperature ($80^\circ \text{C}$), and the control system maintained the plates at this temperature until steady state conditions (steady temperatures and heat input) were reached. Generally, it took about 5 min for the system to reach steady state after a change in the fan operating point, and slightly longer upon the initial startup.

This first measurement point was used to determine the heat loss from the system. With no air flow between the stators or rotation of the impeller, all of the power input from the heaters left the system in the form of heat loss. Since the plates remained at the same temperature for all of the operating points, the heat loss was assumed to be constant throughout each test.

Next, the procedure of the fan curve test was followed, as described in Section 2.3. In the heat transfer characterization, fewer rotational speeds (5) and throttle positions (5) were characterized due to the much longer equilibration times of the thermal system. At each operating point, the fan parameters and also the thermal power input for each plate was recorded for 5 min and averaged.

2.5. Impeller geometries tested

Several impeller geometries, shown to scale in Fig. 5, were characterized. The impellers all have prismatic (i.e. 2-D) geometry that can be fabricated by processes such as waterjet cutting, laser cutting, wire EDM, or punching. This 2-D, planar geometry is a natural choice for planar heat pipes since the condensers between which the impellers rotate are typically flat. The impellers were first waterjet cut from 6061-T6 aluminum sheet. After waterjet cutting and deburring, the center holes were reamed to fit the shaft size (6 mm), and the impellers were flattened with a thermal stress relieving process.

According to the Euler turbomachinery equation, the exit angle and outer diameter should be important geometrical parameters. Dimensional analysis also suggests that the fill ratio and inlet ratio (Section 3.1) should be significant. Therefore, these parameters were varied in the experimentally characterized impellers.

First, an impeller that was developed by Allison [22] was adopted as a performance baseline against which other impellers could be compared. This impeller, referred to as “Fan 1”, was originally devised for a similar heat sink; its design was based on CFD and experiments at the free delivery condition. The blade was backsweped at an angle that changed linearly from 90° at its root to 42° at its tip. Fans 2 and 3 were 5-bladed, constant-width and
had exit angles of 45° and 70°, respectively. Fans 5, 6, and 7 had radial (90°) blades with 15, 10, and 5 blades. For structural reasons their blades had a very slight linear taper, becoming thinner toward the blade tip. Fans 8 (15-bladed) and 10 (20-bladed) also had radial blades but contained an open inner region, with 3 spokes emanating from the center shaft hole and connecting to a support ring. The support ring kept the inlet eye and stator gap entry unobstructed and also improved the rigidity of the impeller blades. The support ring feature proved to be important in multi-layer stacks of fans, to allow air to pass through the inlet eye and avoid starvation of the layers in the center of the stack. Finally, fans 11, 12, 13, and 16 had 15 backswept blades originating from the support ring. Their blades followed a logarithmic spiral and had a thickness distribution roughly proportional to $r^2$, in order to reduce the centrifugal bending force. Their blade angles were 75°, 60°, 45°, and 30°, respectively.

### 3. Results and discussion

#### 3.1. Dimensionless groups

The basic experimental measurements were formed into dimensionless groups to make the results more general and compact. First, the volume flow ($V$) was nondimensionalized by a characteristic velocity (the tip speed, $\omega d_2/2$) and a reference area (the exit plane area, $\pi d_b b_g$), forming the flow coefficient:

$$\phi = \frac{V}{\pi \omega d_b d^2_2 / 2} \quad \text{Flow Coefficient.} \quad (1)$$

Similarly, the pressure rise was nondimensionalized by comparing it to a reference pressure. This reference pressure simply takes the form of the dynamic pressure ($\rho v^2/2$) associated with the tip speed $\omega d_2/2$. For mathematical convenience, however, the $1/2$ is omitted from this pressure. Accordingly, the total-to-static head coefficient is:

$$\psi_{ts} = \frac{\Delta p_{ts}}{\rho \omega^2 d^2_2 / 4} \quad \text{Total – to – static Head Coefficient.} \quad (2)$$

In this head coefficient, the total-to-static pressure rise ($\Delta p_{ts}$) of the impeller is the quantity of interest; that is, the difference between the static pressure at the exit of the impeller and the total pressure (i.e. the sum of the static pressure and the dynamic pressure $\rho v^2/2$, where $v$ is the local velocity) at the inlet to the impeller. Epple discusses the utility of using total-to-static metrics for impeller rating in great detail [27]; essentially these quantities best represent the practical use of a fan and are more universally descriptive of an impeller’s performance. It is worth mentioning that an unsubscripted head coefficient ($\psi$) generally refers to the “total-to-total” head coefficient, meaning that the total pressure at both the inlet and outlet is referenced. If it is anticipated that the exit dynamic pressure can be mostly recovered (e.g. in staged turbomachinery), the total-to-total head coefficient may be of more interest. Japikse and Baines discuss the merits of these various head coefficients in more detail [28].

Next, the power coefficient compares the mechanical power input to the impeller ($W$) to a reference power, the product of the reference volume flow ($\pi \omega d_b d^2_2 / 2$) and the reference pressure rise ($\rho \omega^2 d^2_2 / 4$):

$$\zeta = \frac{W}{\pi \rho \omega^2 b_b d^2_2 / 8} \quad \text{Power Coefficient.} \quad (3)$$

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1. Some authors choose to retain the $1/2$, so some care must be taken when comparing the head coefficient to other publications. This work follows the convention used by Epple [27], which permits another interpretation based on the velocity triangle: the total head coefficient is a ratio of the outlet tangential velocity to the blade tip speed, and the total-to-static head coefficient is analogously defined but with the total-to-static pressure replacing the total pressure.
Three parameters are related to the thermal behavior of the system. First, the Prandtl number characterizes the ratio of momentum diffusivity to thermal diffusivity ($Pr = c_p \cdot \mu / k$); this was not varied in the present work, which focused on air cooled heat sinks over a small temperature range. Second, the coefficient of performance simply compares the heat transfer rate to the mechanical power input ($COP = Q / W$). The COP is often greater than 1; it is not an efficiency, but merely a dimensionless expression of the heat transfer per unit work input in a heat pump. The third thermal parameter is the mean dimensionless heat flux:

$$\Phi_m = \frac{Q}{d_2 \Delta T_{in} k}$$

**Dimensionless Heat Flux.**

where $\Delta T_{in} = T_w - T_m$.

The dimensionless heat flux quantifies the heat transfer conductance (i.e. the heat transfer rate per temperature difference) or resistance of the system. Often, the Nusselt number serves this purpose, but a subtle difference warrants a change in nomenclature from $Nu_m$ to $\Phi_m$ (a detailed discussion of the differences in the behaviors of these two dimensionless parameters can be found in Shah and London [29]). The temperature difference in $\Phi_m$ is between the wall temperature and the inlet fluid temperature. In contrast, $Nu_m$ generally uses the logarithmic mean temperature difference:

$$\Delta T_{in} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

where $\Delta T_1$ and $\Delta T_2$ are the wall-to-fluid temperature difference at the inlet and outlet, respectively. The outlet temperature, of course, depends on the heat transfer into the system, the heat transfer conductance of the system, and the capacity flow rate of fluid through the system. Thus, the temperature scale in $Nu_m$ is seldom known a priori, whereas in $\Phi_m$ the temperature scale is simply $\Delta T_1$ (the inlet temperature difference, often subscripted “ITD”), which is often a prescribed design parameter. From a rating perspective, $Nu_m$ is satisfactory, but for heat sink design purposes $\Phi_m$ can simplify the problem considerably.

Several other dimensionless parameters warrant discussion, although the parameters above are sufficient to describe the problem. First, the efficiency of the fan ($\eta_f$) is a useful metric that compares the power delivered to the air ($V \Delta p_{in}$) to the mechanical power input:

$$\eta_f = \frac{V \Delta p_{in}}{W}$$

**Fan Efficiency.**

Again, the total-to-static quantities are used for consistency. The efficiency is defined in terms of the total-to-static pressure difference because (1) this pressure difference is directly measured in the experimental apparatus, and (2) the total-to-static efficiency is known to be a useful performance metric in centrifugal fan design [27]. Since the flow, head, and power coefficients were defined with a consistent reference area and characteristic velocity, the fan efficiency can also be expressed as

$$\eta_f = \frac{\phi \psi_m}{\zeta}$$

In addition to the exit angle ($\beta_2$) and the number of blades ($Z$), three geometrical ratios are relevant in describing the integrated fan geometry and are referred to by shortened names for convenience. First, the aspect ratio (AR) is defined as $AR = b_0 / d_2$, the ratio of the blade breadth to the outer diameter. Next, the fill ratio (FR) is a measure of how much the impeller blade fills the parallel plate gap, namely $FR = b_0 / h_b$. Finally, the inlet ratio (IR) is the ratio of the stator inner diameter ($d_1$) to the outer diameter ($d_2$); $IR = d_1 / d_2$. As shown in Fig. 3, the stator and impeller have the same outer diameter.

To summarize, the primary dimensionless groups (excluding geometrical ratios) are as follows: $\phi$, $\psi_m$, $\zeta$, and $\Phi_m$. In Section 3.2.3, correlations to estimate the fan performance — that is, the fan curve and power curve ($\psi_m$, $\zeta$ as a function of $\phi$, respectively) — are developed. In Section 3.3.3, a correlation for the dimensionless heat flux is developed. With these correlations, the performance of an integrated fan heat sink can be estimated based on its geometry.

### 3.2. Fan performance (flow rate, pressure rise, and power consumption)

Fan and power curves were measured on the test apparatus. Their general form was observed to be consistent for most of the impellers tested, and above a critical rotational Reynolds number the fan and power curves are well described by the dimensionless parameters $\phi$, $\psi_m$, and $\zeta$, the flow, head, and power coefficients, respectively. Correlations in terms of these parameters were formed for each experiment, which allowed the dimensionless fan and power curves to be represented with two parameters for each fan geometry (each experiment had about 110 operating points). The correlation parameters describing these fits were compared and some patterns based on the fan geometry became evident. The correlation parameters were related to the fan geometry through some simple relations. From a design perspective, these relations allow for the fan performance to be quickly estimated for an unknown fan geometry.

#### 3.2.1. Fan performance: general features

The fan and power curves were observed to be well approximated by linear fits. The dimensionless fan parameters (the flow coefficient $\phi$, the head coefficient $\psi_m$, and the power coefficient $\zeta$) were found to be effective in collapsing all of these speed-dependent curves onto a single curve. Fig. 6 shows the dimensionless fan and power curves for a single experiment (Fan 5, 15 blades, 90° exit angle, $b_0 = 1.6$ mm, $b_2 = 2.6$ mm). Except for the speeds below 3 krpm, all of the points were well represented by a single line. Thus, a particular integrated fan geometry can be compactly represented with two parameters each for the $\psi_m$ and $\zeta$ curves. This will be discussed in detail in Section 3.2.3.

#### 3.2.2. Fan performance: effects of fan geometry

The fan performance curves were observed to be sensitive to the number of blades ($Z$), the blade exit angle ($\beta_2$), and the fill ratio (FR). For example, Fig. 7 shows experimental results for 5, 10, and 15 bladed radial (90° exit angle) fans for various fill ratios. In Fig. 7, the top row of plots shows the fan performance curves, the middle row shows the power curves, and the bottom row shows the total-to-static efficiency curves. The vertical columns of plots correspond to 5, 10, and 15 blade impellers. Finally, the different symbols represent four fill ratios ranging from 0.44 to 0.78.

Increasing the fill ratio resulted in an upward shift of the fan curves, which indicates improved performance. Increasing the number of blades was also beneficial, although this effect diminishes as more blades are added. The fan curves maintain their linear form over these significant ranges in the fill ratio and number of blades.

2 Recalling that the size of the relative eddy (a loss mechanism associated with imperfect guiding of the flow) is related to the width of the blade passage at the exit, it makes sense that the improvement associated with adding blades would diminish; the pitch between blades at the exit plane does not change as much going from 10 to 15 blades as it does going from 5 to 10.
In addition to the fan curves, the power curves are also affected by the number of blades. Using Eq. (7), the flow, head and power coefficients can be combined at each point to calculate the total-to-static efficiency ($\eta_{ts} = \phi_{ts}/\phi$). The efficiency curves can be seen to improve with increased fill ratio and number of blades; however, the difference in efficiency between the 10- and 15-bladed fans appears to be negligibly small. This may be an indication that the frictional losses in the blade passage begin to outweigh the efficiency improvement associated with better flow guidance. CFD simulations discussed in Chapter 3 of Staats [26] suggested an optimum number of blades around $Z = 17$.

The fan curve also depends on the blade angle at the exit plane. This dependence makes sense because the blade angle appears in the Euler turbomachinery equation and therefore changes the theoretical maximum fan performance curve that can be reached. The blade angle affects the relationship between the meridional and...
tangential velocity components, which influences the slip and the frictional losses. The trends associated with variation of the blade angle, based on the Euler turbomachinery equation, were observed experimentally; Fig. 8 shows measurements of the total-to-static head coefficient, the power coefficient, and the efficiency for 90°, 60° and 45° exit angle impellers. Three values of the fill ratio are shown in each plot.

As the blade angle is reduced (i.e. as the blades become more backswept), the maximum head coefficient increases. The maximum efficiency increases, although diminishing returns appear evident in comparing the 60° and 45° impellers.

3.2.3. Fan curve correlation

The fan curves were well described by the dimensionless forms of the volume flow and pressure rise (the flow coefficient \( \psi \)) and the total-to-static head coefficient \( \psi_{ts} \). These dimensionless curves exhibited linear behavior (Fig. 6). A linear correlation for the total-to-static head coefficient as a function of the flow coefficient represents the dimensionless fan curves of each experiment with two parameters:

\[
\psi_{ts} = \psi_{ts,max} - C_t \phi,
\]

where \( \psi_{ts,max} \) is the maximum or “blocked” head coefficient and \( C_t \) is a coefficient that represents the slope of the dimensionless fan curve. The maximum flow coefficient can be expressed as a combination of \( C_t \) and \( \psi_{ts} \) as

\[
\phi_{max} = \frac{\psi_{ts,max}}{C_t},
\]

this parameter is convenient because of its physical meaning, which is a dimensionless form of the free delivery flow rate, the flow that is delivered when pumping against a resistance-free system. In general, this linear, dimensionless correlation for the fan curves resulted in accurate estimates of the fan performance. The details of each fan curve experiment, including the fill ratio (FR), the aspect ratio (AR), the inlet ratio (IR), the blade exit angle \( \beta_2 \), the number of blades \( Z \), the maximum head coefficient \( \psi_{ts,max} \), the maximum flow coefficient \( \phi_{max} \), and the fan curve slope \( C_t \) can be seen in Table 4.3 of Staats [26].

The correlation parameters \( \psi_{ts,max} \) and \( \phi_{max} \) themselves were examined to see if any geometry dependence could be ascertained. A correlation related to the geometry of the impellers would be useful in the early phase of a design, when it is desirable to have rough estimates of a hypothetical fan’s performance based on high-level design parameters. A reasonable agreement with the data was obtained by fitting these experiment-specific parameters as follows:

\[
\psi_{ts,max} = FR \left( a_1 + a_2 Z \sin(\beta_2) \right),
\]

\[
\phi_{max} = FR \left( b_1 + b_2 Z^{-1} \right),
\]

where \( a_i \) and \( b_i \) are constants with values shown in Table 1. With this form, both coefficients approach zero as FR approaches zero. The \( \sin(\beta_2) \) term was motivated by the velocity triangle and seemed

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Fan and power curve parameter estimators.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_{max} ) estimator (a, Eq. (10))</td>
<td>0.2257</td>
</tr>
<tr>
<td>( \psi_{ts,max} ) estimator (b, Eq. (11))</td>
<td>0.5970</td>
</tr>
<tr>
<td>( C_t ) estimator (c, Eq. (13))</td>
<td>0.0132</td>
</tr>
<tr>
<td>( C_w ) estimator (f, Eq. (14))</td>
<td>0.2372</td>
</tr>
</tbody>
</table>
are shown in Table 1.

We observed that the power coefficient curve does not change appreciably under heated-stator conditions. We have observed that the y-intercept (shut-off head coefficient) and the free delivery flow coefficient (defined in Eq. (3)) was observed to be linear as mentioned above. \( \xi \) was correlated using a form similar to Eq. (8):

\[
\xi = \xi_0 + C_\xi \phi.
\]

where \( \xi_0 \) is the power coefficient at zero flow coefficient, and \( C_\xi \) is the slope of the dimensionless power curve. The values of \( \xi_0 \) and \( C_\xi \) for each experiment can be seen in Table 4 of Staats [26]; this linear correlation of each fan geometry resulted in 10% or better relative root-mean-squared error in all but 6 of the 73 experiments. Additionally, the correlation parameters \( \xi_0 \) and \( C_\xi \) were observed to have a relationship to the fan geometry:

\[
\xi_0 = c_1 + c_2 FR \sin (\beta_2) \quad \text{and} \quad C_\xi = f_1 FR \sin (\beta_2) Z_1^{1/2}.
\]

Eqs. (13) and (14) allow the two parameters that define the power curve for each fan \( \xi_0 \) and \( C_\xi \) to be estimated based on the fan geometry. The coefficients \( c_1 \) and \( f_1 \) are shown in Table 1. The agreement between Eqs. (13) and (14) and the power curve parameters determined from each experiment is shown in Fig. 9. Most of the power curve parameters are estimated to within 20% by Eqs. (13) and (14).

The data underlying the fan and power curve correlations (Eqs. (8)–(14)) were obtained in 73 experiments, and the salient design variables spanned the following values: 5 \( \leq Z \leq 20 \), 0.10 \( \leq FR \leq 0.86 \), 30 \( \leq \beta_2 \leq 90 \), 0.4 \( \leq IR \leq 0.5 \), and 0.01 \( \leq AR \leq 0.03 \).
3.3. Heat transfer

The average heat transfer coefficient was determined by measuring the heat input ($Q$) required to maintain the stators at a constant temperature ($T_m$). The average heat transfer coefficient is

$$h_{ITD} = \frac{Q}{A_s \Delta T_m},$$

(15)

where $A_s$ is the wetted surface area of the stator plates. The “ITD” subscript stands for “inlet temperature difference”, so as to distinguish this heat transfer coefficient from the conventionally defined average heat transfer coefficient, which references the log-mean temperature difference between the wall and the fluid stream. The inlet temperature difference ($\Delta T_m = T_w - T_m$) was determined by measuring the wall temperature and inlet air temperature with thermocouples, as described in Section 2.2. The high thermal conductivity of the copper stator plates ensured that their temperature remained spatially uniform. The wetted surface area of the stator plates ($A_s$) comprises the two impeller-facing annular faces, namely

$$A_s = 2 \cdot \pi (r_f^2 - r_i^2),$$

(16)

where $r$ is the radius and the subscripts 1 and 2 refer to the core and the impeller blade tip, respectively (Fig. 3).

3.3.1. Heat transfer: general features

The heat transfer coefficient was measured for various rotational speeds and flow rates. The heat transfer coefficient increases with volume flow; the presence of the rotating impeller also causes a significant increase in the heat transfer coefficient. At a given volume flow, $h_{ITD}$ at the highest rotational speed is sometimes three times its value in the absence of rotation.

The heat transfer coefficient can be expressed in dimensionless form as the dimensionless heat flux ($\Phi_m$). As discussed in Section 3.1, $\Phi_m$ and the Nusselt number ($Nu$) have similar forms but, importantly, reference different temperature scales (the inlet and the log-mean temperature differences, respectively).

In the integrated fans, it was observed that using the streamwise meridional flow length $L_f = r_f - r_i$ as the characteristic length in the dimensionless heat flux gave a more consistent representation of the experimental data than did the hydraulic diameter. This modification in the length scale removes the direct dependence on the gap breadth that would exist in $\Phi_m$ had the hydraulic diameter been chosen as the length scale. The mean dimensionless heat flux thus becomes

$$\Phi_{m,L} = \frac{h_{ITD} L_f}{k},$$

(17)

where $k$ is the thermal conductivity of the air and $L_f$ is the radial flow length along the passage $(r_f - r_i)$. The additional “L” in the subscript of $\Phi_{m,L}$ serves as a reminder that the length scale is $L_f$ rather than the hydraulic diameter.

The dimensionless heat flux (heat transfer coefficient) is affected by the volume flow as well as the rotational speed of the impeller. Each of these operating parameters can be represented by a respective Reynolds number. The channel Reynolds number, $Re_m$, is the Reynolds number at the arithmetic mean radius $r_m = (r_f + r_i)/2$, which simplifies to

$$Re_m = \frac{V}{\pi D r_m},$$

(18)

where $v$ is the kinematic viscosity of the air. $Re_m$ has the hydraulic diameter, $d_h = 2b_h$, as its length scale and the average velocity at $r_m$ as its velocity scale. In the heat transfer experiments, the channel Reynolds number proves more convenient than the flow coefficient ($\phi$) because it contains no influence from the rotational speed, allowing for decoupling of the two independently controllable parameters (volume flow and rotational speed). The rotational speed $\omega$ is represented in dimensionless form by the rotational Reynolds number:

$$Re_\omega = \frac{c_0 r_m^2 \omega}{v}.$$

(19)

The dimensionless heat flux was observed to increase linearly with the channel Reynolds number at each rotational speed, as shown in Fig. 10. In each characterization, the fan was held at a constant speed and various flow rates were allowed through the system by varying the throttle. Each group of points (connected by lines) in Fig. 10 represents a constant rotational speed (constant $Re_\omega$). The dimensionless heat flux was also observed to increase linearly with $Re_\omega$.

The heat transfer is insensitive to the planform profile of the impeller. For example, consider 4 fans: impellers 8, 11, 12 and 13, which are very similar except for the backsweep angle of their blades. The dimensionless heat flux for each of these impellers, shown in Fig. 10, has a very similar value at most of the test points, which suggests that the heat transfer depends on the operating point (flow rate and rotational speed) rather than directly on the impeller geometry.

3.3.2. Heat transfer: analytical estimates

3.3.2.1. Surface renewal theory. Surface scraping is commonly used to enhance the heat transfer in viscous liquids. In this method, a scraper moving in close proximity to the wall mechanically removes the fluid adjacent to the wall, allowing fresh bulk-temperature fluid to take its place. Scraping is generally useful in laminar flow, where the boundary layer introduces a significant impediment to the convective heat transfer.

This phenomenon is exploited in some industrial applications such as polymer, pharmaceutical, and asphalt processing [30]. For example, polymer extrusion screws exhibit this behavior in the small clearance space between the screw and the wall. This mechanical enhancement can increase the heat transfer coefficient significantly. The heat transfer coefficient ($h$) for rapid renewal, when the transient conduction penetration depth is thin, is

![Fig. 10.](image-url)
\[ h = 2 \sqrt{\frac{\nu \rho c_p}{\pi} \sqrt{f}}. \]  

where \( f \) is the frequency of surface renewal. This model of the convective enhancement assumes that the fluid near the surface is periodically wiped away and instantly replaced with bulk-temperature fluid, and is referred to as “penetration theory” or “surface renewal theory” in the literature \([31,32]\).

The experimental data shown in Fig. 10 can be estimated by the surface renewal theory. However, one key trend predicted by the surface renewal theory was not observed experimentally. A series of tests were performed on impellers 5–7, which were identical except for the number of blades. Eq. (20) predicts that the heat transfer should scale with the square root of the renewal frequency (i.e. the blade-passage frequency), which is related to the rotational speed \( (\omega) \) and number of blades \( (Z) \) as \( f = Z\omega/(2 \cdot \pi) \). The heat transfer coefficient was calculated, and the total heat transfer determined using the effectiveness-\( Nu \) method. The inlet-temperature-difference heat transfer coefficient was calculated from this overall heat transfer. The measured heat transfer coefficients are barely different for analogous tests with impellers 5–7; this is in contrast to the surface renewal theory estimates, which are significantly different. The surface renewal theory correctly suggests an enhancement due to increases in rotational speed, but does not appropriately estimate the effect of additional blades. Additionally, the surface renewal theory significantly underestimates the heat transfer at low channel Reynolds numbers (low volume flow rates). This is likely due to an unmodeled recirculatory flow at the exit plane of the impeller.

When the inlet is blocked and the net flow through the fan is zero, the relative eddy dominates the flow in the blade passage. This eddy, at the exit plane, results in some reentry flow that is likely to have a temperature close to ambient, resulting in a significant cooling effect. The surface renewal theory results above were modeled under the assumption that the air flows unidirectionally through the passage from inlet to outlet, so of course this recirculation effect did not manifest itself in the above-discussed estimate.

These findings are similar to those of Hagge and Junkhan \([32]\), who studied external flow over a flat plate fitted with a rotating scraper blade. They found that the surface renewal theory was applicable at low freestream Reynolds numbers, and served to estimate the general trends of the heat transfer; however, they also reported that the surface renewal theory was not capable of standing alone as a predictive tool for design purposes. They suggested that the main heat transfer mechanism was instead the turbulent flow established between the blade and the heated wall.

3.3.2.2. Turbulent flow. Another approach to modeling the heat transfer in an integrated fan heat sink is to assume that the flow between the blade and the heated wall is turbulent, and calculate the heat transfer using turbulent pipe flow correlations and the effectiveness-\(Nu\) method. Several Reynolds numbers were tested in this approach since the appropriate choice of length and velocity scale was unclear. Using a Reynolds number based on an “effective velocity” similar to that of Hagge and Junkhan \([32]\) and a length scale of the hydraulic diameter, reasonable agreement with the experimental data at higher flow rates was achieved. The “effective velocity” used by Hagge and Junkhan was the sum of the blade speed and the flow speed. In the present work, an analogous internal flow approach was taken, using an effective velocity calculated (as a function of radius \(r\)) as the sum of the meridional velocity \(c_m\) and the local blade speed \(\nu_{bl}\):

\[ \nu_{eff}(r) = u(r) + c_m(r) = \nu_{bl} + \frac{V}{2\pi rb}. \]  

The effective Reynolds number used \(\nu_{eff}\) as the velocity scale and the hydraulic diameter as the length scale: \(Re_{eff} = \rho \nu_{eff} db/\mu\). The Nusselt number was then calculated using a turbulent flat plate correlation as was done by Hagge and Junkhan:

\[ Nu = 0.0249 Re_{eff}^{0.8}. \]  

\(Nu\) is the local Nusselt number for flow over a flat plate, defined as \(Nu = h x / k\), where \(x\) is the distance from the leading (entry) edge of the plate and \(h\) is the local heat transfer coefficient, referenced to the local wall-to-bulk temperature difference. The average heat transfer coefficient was determined by integrating Eq. (22) over the length of the flow channel (accounting for the changing \(Re_{eff}\)) and dividing by the length. The effectiveness-\(Nu\) method was used to calculate the total heat transfer while accounting for the temperature rise of the air stream. The total heat transfer was then used to calculate the inlet-temperature-difference heat transfer coefficient \(h_{ITD}\).

Fig. 11 shows the results of this analysis in comparison with experimental data (for Fan 1, a 5-bladed impeller with \(b_r = 2.5\) mm and \(b_t = 1.5\) mm). Hagge and Junkhan’s turbulent heat transfer approach accounts for the enhancement due to faster rotational speeds. It predicts no improvement in the heat transfer with the addition of blades, contrary to the surface renewal theory, and is more representative of the experimental observations.

Also shown in Fig. 11 are two analytical estimates of the heat transfer from the radially outward flow when the impeller is stationary. Suryanarayana’s correlation \([33]\) for radially outward flow is shown to slightly underestimate the heat transfer. Additionally, for reference, a correlation for simultaneously developing laminar flow (hydrodynamic and thermal boundary layer development in concert) is also shown (given in Ebadian and Dong \([34]\) for parallel plates with constant temperature walls). The entry length has a higher heat transfer coefficient than in the fully developed regime; even accounting for this, the estimated heat transfer is much lower than the experimentally observed heat transfer in the presence of rotation, further supporting the hypotheses that (1) the fan causes turbulence in the channel despite the relatively low through-flow velocities and (2) recirculatory flow at the exit plane causes higher heat transfer at low volume flow rates.

![Fig. 11](image-url)
Of these methods, the turbulent flow approach (the internal flow analog of Hage and Junkhan’s method) seems to have the most merit, in that it (1) correctly models enhancement of the heat transfer with higher rotational speed, (2) predicts no significant enhancement with addition of impeller blades, and (3) comes reasonably close to the experimentally observed heat transfers at higher flow rates, where recirculation is likely negligible. The hypothesis that the convection enhancement arises from impeller-induced turbulent flow structures is also supported in the local heat transfer measurements of Staats [26], which were performed for constant-heat-flux and constant-temperature boundary conditions and yielded no discernible difference in the heat transfer coefficient. This insensitivity to the thermal boundary condition is suggestive of turbulent flow.

As with the surface renewal theory results, the main failure of these turbulent flow estimates occurs at the lower volume flows; the assumption of unidirectional through-flow inherent to these models was observed to be untrue. An attempt was made to correct for this by modeling the outlet recirculation in a manner similar to that of Qiu et al. [35]. Ultimately, this approach did not satisfactorily match the experimental data without correction factors specific to each impeller geometry. Rather than supplementing these analytical methods with correction factors to capture the system’s behavior, an experimentally based approach using dimensionless parameters was pursued, leading to a more accurate and simpler reduction of the experimental data.

3.3.3. Heat transfer: correlations

Experimentally based correlations were developed to allow for the estimation of the heat transfer in an integrated fan with knowledge of the aspect ratio (AR = bL/dh) and operating point (i.e. the flow rate and rotational speed). A key finding was that the impeller profile and fill ratio (i.e. the fan geometry) did not have a significant, direct effect on the heat transfer, and therefore the heat transfer problem is largely independent of the fan design problem. This is a convenient result because simple fan characterization tests are much easier to perform than the comparatively laborious, complicated, and expensive heat transfer experiments. The insensitivity of the heat transfer performance to the fan geometry implies that a designer could quickly and efficiently characterize a new design on a relatively simple apparatus and subsequently estimate its heat transfer with confidence using the correlations developed in this section. Generally speaking, an efficient, well designed integrated fan will also excel in terms of heat transfer performance.

It is worth clarifying that the insensitivity of the heat transfer to the impeller blade profile does not suggest that the impeller profile is unimportant in the design process. On the contrary, the impeller profile has a significant effect on the system performance, because it has a strong effect on the fan and power curves. A well designed fan will pump more air through a given system and result in an operating point with a higher flow rate compared to a poorly designed fan operating at the same speed. Thus, since the impeller blade profile determines the operating point, the blade profile indirectly affects the heat transfer performance. But as was shown in Fig. 10, the direct dependence of the heat transfer on the impeller geometry is weak. The heat transfer mostly depends on the flow rate and rotational speed.

A two-dimensional linear correlation for the dimensionless heat flux (Φm,l) in terms of the two Reynolds numbers in Eqs. (18) and (19) was formed to fit the data from each experiment. These correlations had the form

$$\Phi_{m,l} = C_1 \Re_{\alpha} + C_2 \Re_{\alpha h},$$

where C1 and C2 were determined through a linear least squares minimization. Φm,l is a dimensionless heat flux that uses the flow length along the plate (Lt = R2 − R1) as its length scale, defined in Eq. (17).

The relative root-mean-squared error (rRMSE) was used as the error metric of the correlation fit and is defined as

$$\text{rRMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( \frac{x_{m,l} - \hat{x}_{m,l}}{x_{m,l}} \right)^2},$$

where xmi is the measured quantity and x̂mi is the correlation-estimated quantity for data point i of an experiment with n data points. The linear fit proposed in Eq. (23) has small values of rRMSE (typically less than 10%) for each experiment, meaning that this form of correlation represents each data set with excellent fidelity.

Next, the experimental data was collapsed even more by developing an estimator for the correlation coefficients C1 and C2 in Eq. (23). An average value of C1 and C2 could be taken over all experiments, but first the distributions of C1 and C2 were examined to determine whether they were normally distributed. Both C1 and C2 seemed to have relatively normal distributions, except for a few errant data points. These data points corresponded to experiments with larger aspect ratios than the majority of the data.

To improve the estimate of C1 and C2, equations were formed involving the aspect ratio. These estimation methods must be used cautiously because a relatively small amount of experimental data at larger aspect ratios was collected (5 experiments had AR = 0.031, while the remainder of the 29 experiments had AR ≈ 0.015). Distributions of C1/AR1/2 and C2/AR1/2 seemed to be much more normal with the modification associated with the aspect ratio, based on the linearity of their normal probability plots (also called “Q-Q” plots; see Wilk and Gnanadesikan [36] for greater detail) and the shape of their histograms. Taking the mean of this remapped data leads to equations that can estimate C1 and C2 for any of the characterized fans. These estimates were as follows:

$$C_1 = 0.0044 \cdot AR^{1/2}$$

$$C_2 = 0.0065 \cdot AR^{-1/2}.$$  

(25)

(26)

These equations for C1 and C2 can be substituted into Eq. (23), yielding

$$\Phi_{m,l} = 0.0044 \cdot AR^{1/2} \Re_{\alpha} + 0.0065 \cdot AR^{-1/2} \Re_{\alpha h}.$$  

(27)

which is a global correlation for the dimensionless heat flux that spans all of the experimental data in 29 heat transfer experiments, with the following ranges: 0 ≤ Reα ≤ 1.53 × 105, 0 ≤ Reαh ≤ 4.72 × 105, 0.015 ≤ AR ≤ 0.031, and 0.33 ≤ Fr ≤ 0.90. All of the heat transfer experiments had the same inlet ratio (Ir = 0.4).

These correlations for C1 and C2, of course, result in additional error in reproducing the original experimental data. Still, using Eq. (27), the dimensionless heat flux can generally be estimated to within 20%, as can be seen in Fig. 12. If fan geometry happens to correspond to one of the fans characterized in the present work, the coefficients C1 and C2 can be directly obtained from Table 4.3 of Staats [26], which results in a more accurate (less than 13% rRMSE) estimate of Φm,l compared to the estimate produced by Eq. (27) (typically less than 20% rRMSE), at the expense of reduced generality.

In summary, the heat transfer for an integrated fan is well approximated by a two-variable linear correlation for the dimensionless heat flux (Eq. (23)). The constants C1 and C2 can either be determined by consulting Table 4.3 of Staats [26] or using Eq. (25), which is a function of the aspect ratio of the impeller.
3.4. Thermo-mechanical coupling

In the previous sections, the effects of the fan geometry on the pumping, power consumption, and heat transfer were shown. Considering the thermal performance at the same time as the fan performance reveals an interesting relationship. Fig. 13 shows the thermal resistance and mechanical input power for several fans. Each plot shows the effect of one geometrical parameter by comparing 3 fans. The leftmost plot shows the effect of fill ratio (FR), the center plot shows the effect of number of blades (Z), and the rightmost plot shows the effect of the exit blade angle (β₂). On each plot, groups of constant rotational speed are connected by lines, with the higher speeds on the right.

The thermal resistance vs. mechanical power plot can easily lead to misinterpretations about the thermo-mechanical performance of the fan. For example, at first glance, the center plot seems to indicate that the 5-blade fan performs better than the 10- and 15-blade fans, because it has a lower thermal resistance at the same speed and constant mechanical power (e.g., at about 6 W, the 5-blade trace has a thermal resistance of 0.46 K/W while the 15-blade trace has a thermal resistance of 0.55 K/W). However, the fans with more blades can reach a lower thermal resistance at a slower speed compared to the 5-blade fan. The rightmost point on each connected series of constant-speed points represents the highest flow rate measured at that speed. Again referring to the center plot, the 10- and 15-bladed impellers were capable of reaching a higher flow rate at a given speed, so their curves extend farther right and allow lower thermal resistances to be achieved. The solid black lines near the bottom of each data set is the best-performance frontier (also known as a Pareto frontier) based on all of the data observed in the test apparatus.

A fan’s operating range consists of the entire envelope of points shown on the R vs. W plot since it can operate at speeds between those shown in Fig. 13. The best points are the lowest thermal resistance at a given mechanical power input. The bounding envelope of each data set looked similar, because small changes in the heat transfer appear as even smaller changes in R. To better compare the best operating points, all of the heat transfer data were viewed in a different representation of the fan-heat-transfer coupling. Fig. 14, in which the mechanical power (W) is plotted as a function of the average heat transfer coefficient (h_{ITD}), shows a distinct Pareto frontier of best-performing points. This frontier was found to be well represented by a power law fit for W in terms of h_{ITD}, namely

\[ W = m_1 h_{ITD}^{\frac{1}{2}}. \]  

where the constant \( m_1 \) has a value of \( 6.04 \times 10^{-9} \) m² K/W. This dimensional representation, although not as elegant as a dimensionless approach, used a very minimal amount of data processing prior to fitting; \( h_{ITD} \) and \( W \) are essentially direct measurements. The curve defined by Eq. (28) is shown as a solid black line in Fig. 14. Fan 12 (15 blades, 60° exit angle) appears to occupy many points along the frontier, suggesting that Fan 12 is one of the best designs studied in the present work.

The frontier of best operating points can be expressed in terms of thermal resistance by manipulating Eq. (28) using the relationship between R and \( h_{ITD} \) (viz. \( R = 1/(h_{ITD} \cdot A_s) \)):

\[ R = m_1^{-1} W^{-1/4} A_s^{-1}. \]  

where \( A_s \) is the heated surface area of the stator gap (defined in Eq. (16)). Eqs. (28) and (29) can be used as a best-case estimate of the achievable thermal performance of a single integrated fan (of the
100 mm diameter size explored in this work) given a mechanical power constraint. For reference, the DARPA MACE goal sought a thermal resistance of 0.05 K/W with an electrical power input of 33 W [3]. Assuming that 33 W of mechanical power was available for the fan, a single integrated fan could only reach a thermal resistance of 0.275 K/W according to Eq. (29). In Chapter 5 of Staats [26], it was shown that a simple solution to this apparent limitation is to use multiple parallel integrated fans in a single device. The thermal performance of these multilayer heat sinks can be much lower than Eq. (29) would suggest.

Some prior work on the heat transfer in integrated fans similar to the present work was reported by Allison [22,23], who gives two equations for R representing the best design points (equations use Sl units):

\[
R = 0.387 W^{-0.333} \quad \text{(Exp. Measurements [22])} \tag{30}
\]

\[
R = 0.338 W^{-0.643} + 0.373 \quad \text{(CFD [22])} \tag{31}
\]

For comparison, in this work, Eq. (29) evaluates to \(R = 0.668 W^{-0.25}\). Both the experimental measurements and CFD simulations conducted by Allison focused only on the free delivery point. Since the free delivery point was observed to be the best operating point in the present work, Allison’s results should be consistent. However, they were observed to be substantially lower in thermal resistance. Several causes could explain this discrepancy. First, the heat losses in Allison’s experiments were only estimated rather than experimentally characterized, which could have led to underestimates of the heat loss (in this scenario, some of the lost energy due to conduction through the insulation, for example, would be counted as part of the reported convective heat transfer). Second, the bottom heated plate extends all the way to the impeller shaft, where there is some additional heat transfer surface area that was possibly unaccounted for on the bottom plate within the core. The heat transfer in this region could be significant, because the flow entering the eye impinges upon the bottom plate and results in thin boundary layers with high heat transfer coefficients. In the present work, the core region had an identical hole in the top and bottom plates, leading to lower heat transfer. The optimal design frontiers reported by Allison are compared to Eq. (29) in Fig. 15. Allison’s CFD-computed frontier shows reasonably good agreement with the present experimental data while his experimental-measurement-based correlation for the frontier estimates substantially lower thermal resistances.

Referring back to Fig. 13, the frontier (Eq. (29)) seems to be close to the experimental data, although some points approach the frontier more closely than others. Notably, the 60° and 45° exit angle blades in the rightmost plot come very close to the frontier. In fact, the 60° points shown in Fig. 13 correspond to Fan 12, which was also identified as one of the best designs in Fig. 14. In addition to showing the best geometrical designs, the best \textit{operating points} for a given design can be identified in Fig. 13. The points with the least flow restriction (the rightmost points on each line of constant speed) approach the frontier more closely than the restricted points, confirming that operating with minimal flow restriction (highest flow coefficient) is preferable to achieve the best performance. Although this may seem obvious in hindsight, these points do not represent the best efficiency point of the fan; the best efficiency point occurs when the fan operates at a lower flow coefficient (Fig. 8). This demonstrates the sometimes-counterintuitive nature of integrated fan heat sink design. Conventional fan design can be unsuitable for the design of integrated fan heat sinks because, unlike a conventional fan, the central figure of merit may not be related to the fan performance at all. In this case, an operating point that is suboptimal for fan efficiency yields the best heat transfer performance.

4. Conclusions

In this work, we present experimental results (fan pumping capability, fan power consumption, and heat transfer) of prototype “integrated fan heat sinks”, which consist of a fully unshrouded centrifugal fan interdigitated between two heated stators. This air flow topology is complementary to newly developed planar heat pipes; a concept for a loop heat pipe with multiple parallel
condensers and integrated fans offers the potential to achieve very low thermal resistance (\( R < 0.05 \text{ K/W} \)) in a small volume (\( V < 1 \text{ L} \)). The main conclusions of the present work are as follows:

1. The fan performance (specifically, the pressure rise and mechanical power consumption) was correlated with a pair of 2-parameter correlations for each of 73 fan performance experiments. These correlations (for the head and power coefficients as functions of the flow coefficient) estimated the data for each experiment with a relative RMS error of less than 19% for the pumping and 27% for the mechanical power (within 95% confidence intervals). The correlation parameters were then expressed as functions of the fan geometry; these functions could estimate the correlation parameters to within 20%, and serve as high-level design tools for integrated fans.

2. The heat transfer in a single integrated fan was experimentally characterized for 12 different impeller profiles with various impeller thicknesses and stator gap spacings. The dimensionless heat flux was observed to increase linearly with both the flow Reynolds number and the rotational Reynolds number. Based on this observation, a two-parameter correlation was developed for each experiment that predicts the data to within a relative RMS error of less than 10% (with a 95% confidence interval). A global correlation for the heat transfer was also developed and fits the majority of the data from the 29 heat transfer experiments to within 20%.

3. The convective heat transfer in an integrated fan heat sink exhibits a marked enhancement compared to a pressure driven flow at a similar flow Reynolds number, due to the presence of the rotating impeller in the flow passage. This heat transfer enhancement can be ascribed to turbulent flow structures induced by the impeller. The number of blades had only a minor effect on the heat transfer, suggesting that surface renewal theory (i.e. mechanical boundary layer disruption) does not adequately explain the heat transfer enhancement.

4. Estimating the heat transfer in integrated fan heat sinks can be done separately from the fan design problem. The fan design (which can be completed assuming isothermal conditions) determines the operating point in a system, and the heat transfer is a function of this operating point (not of the basic fan geometry, except for the aspect ratio of the impeller). This separability of the fan and heat transfer performance greatly simplifies the design of integrated fan heat sinks because the fan analysis and characterization (either from experiments or CFD) is much less complex under isothermal conditions.

5. A tradeoff between thermal resistance and pumping power was observed, along with a Pareto frontier representing the best design points. Designs that minimized the thermal resistance at a given pumping power approached this optimal frontier. For a particular fan design, the operating points that most closely approached the optimal frontier were the points with the highest volume flow (least flow restriction) at each rotational speed (i.e. the free delivery point). These optimal operating points did not correspond to the points of highest fan efficiency, serving as a reminder that integrated fan heat sink design has different goals than conventional fan design.

Conflict of interest
None declared.

Acknowledgments
This work was supported by the DARPA Microtechnologies for Air-Cooled Exchangers (MACE) program, grant \# W31P4Q-09-1-0007, under program management from Drs. Thomas Kenny and Avram Bar-Cohen. The authors thank our colleagues, Professors Evelyn Wang and Jeffrey Lang. We thank Professor Philipp Epple for providing a copy of his doctoral dissertation. Additionally, we thank Ari Umans, Tess Saxton-Fox, and Kristyn Kadala for assistance with the experimental data collection.

References

